Modified-Dewan Optical Turbulence Parameterizations

Artie Jackson

6 June 2004

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1 INTRODUCTION

The Dewan optical turbulence parameterization (Dewan et al. 1993) has been the Air Force Research Laboratory optical turbulence model of choice for various research efforts involving optical propagation during the past several years. The Dewan parameterization was developed to convert standard radiosonde data into vertical profiles of C_n^2 , the refractive index structure constant, which is the critical parameter for describing optical turbulence. The Dewan parameterization is also being used to forecast optical seeing conditions for ground-based telescopes at the Mauna Kea Observatories on the Island of Hawaii (Businger et al. 2002) by converting standard Numerical Weather Prediction (NWP) forecast model output into vertical profiles of forecast C_n^2 .

The Dewan parameterization provides useful vertical profiles of $C_n^{\ 2}$ in the upper troposphere and stratosphere, though there is certainly a need for improvement at these altitudes. The statistical relationships relating Dewan's Y parameter to wind shear, which form the basis of the Dewan parameterization, are often not found in atmospheric measurement data, thus this is an area to explore in developing improved optical turbulence parameterizations for the troposphere and stratosphere. The Dewan parameterization was not developed for use in the lower troposphere. Several modified-Dewan parameterizations have been developed for use in the lower troposphere.

2 AFRL OPTICAL TURBULENCE MODELS

2.1 Dewan

The Dewan optical turbulence parameterization (Dewan et al. 1993) was developed

to convert standard radiosonde data into vertical profiles of C_n^2 at 300 m vertical resolution. The Dewan parameterization uses the Tatarski (1961) formulation for C_n^2 ,

$$C_n^2 = 2.8 \left(\frac{(79x10^{-6}P)}{T^2} \right)^2 L_o^{4/3} \left(\frac{\partial T}{\partial Z} + \gamma_d \right)^2$$
 (1)

where P is the pressure in mb, T is the temperature in K, γ_d is the dry adiabatic lapse rate of 9.8×10^{-3} °K m⁻¹, and Z is the height in m. These parameters are available in standard radiosonde data. $L_o^{4/3}$ is defined as the outer scale of turbulence, in m. Dewan developed a statistical relationship for $L_o^{4/3}$ as a function of wind shear, modifying Equation (1) to

$$C_n^2 = 2.8 \left(\frac{\left(79x10^{-6}P\right)}{T^2} \right)^2 (0.1)^{4/3} \left(\frac{\partial T}{\partial Z} + \gamma_d \right)^2 10^{\gamma}$$
 (2)

where the Y parameter is a linear function of wind shear. Dewan developed separate troposphere and stratosphere relationships for the Y parameter. The relationships for Y as a linear function of wind shear are:

$$Y = 1.64 + 42.0 \text{ x Shear}$$
 (Troposphere) (3a)

$$Y = .506 + 50.0 \times Shear$$
 (Stratosphere) (3b)

where Shear has units s⁻¹.

2.2 CLEAR1

The CLEAR1 optical turbulence parameterization (Beland 1993) provides an artificially smooth, though reasonably typical average nighttime C_n^2 profile from the ground to 30 km. The CLEAR1 C_n^2 profile is a statistically derived function of height only, thus does not require any meteorological input. The parameterization has been

used as the baseline for expressing the optical turbulence design criteria for optical systems.

The CLEAR1 parameterization is composed of three statistical relationships representing three height ranges. The relationships and coefficients are:

$$\log_{10}(C_n^2) = A + Bz + Cz^2 \qquad 1.23 \le z \le 2.13 \tag{4}$$

where A = -10.7025, B = -4.3507, and C = 0.8141,

$$\log_{10}(C_n^2) = A + Bz + Cz^2 \qquad 2.13 < z \le 10.34$$
 (5)

where A = -16.2897, B = 0.0335, and C = -0.0134, and

$$\log_{10}(C_n^2) = A + Bz + Cz^2 + De^{-0.5\left(\frac{z-E}{F}\right)^2}$$
 10.34 < z \le 30.00

where A = -17.0577, B = -0.0449, C = -0.0005, D = 0.6181, E = 15.5617, and F = 3.4666. For the CLEAR1 parameterization, z is the height above the ground in km.

The modified-Dewan parameterization C_n^2 profiles developed in this study are verified against observed C_n^2 profiles and compared to the CLEAR1 and Dewan parameterization C_n^2 profiles to determine their potential to provide improved optical turbulence profiles.

3 OPTICAL TURBULENCE DATA SETS

The data sets used in this study were derived from the Air Force Research Laboratory (AFRL) Holloman (New Mexico) Spring 1998 and Holloman Spring 1999 thermosonde campaigns. There are 20 profiles with continuous data from 1.572 kilometers to 29.772 kilometers from the Holloman Spring 1999 thermosonde campaign that were used to develop modified-Dewan parameterizations and 8 profiles with

continuous data from the Holloman Spring 1998 thermosonde campaign that were used to verify parameterizations. The balloon-borne thermosonde instrument, described in Brown et al. (1982), which is attached to a standard meteorological radiosonde, provides measurements of C_T^2 , the temperature structure function. Jumper and Beland (2000) describe how C_n^2 , the refractive index structure constant, is derived from the thermosonde measurements. Thus, each profile contains optical turbulence and standard meteorological measurements.

An additional parameter, Y_{OBS}, the value of Dewan's Y parameter calculated for a particular set of atmospheric conditions, is required for model development and verification. Using Dewan's formulation for C_n² shown in Equation (2) and solving for the Y parameter, then substituting observed meteorological data (pressure, temperature and temperature lapse rate) and coincident thermosonde data (C_n²), the value of the Y parameter can be calculated for a particular set of atmospheric data.

$$Y_{OBS} = \log \left\{ C_n^2 / \left[2.8 \left(\frac{(79x10^{-6}P)}{T^2} \right)^2 \left(\frac{\partial T}{\partial Z} + \gamma_d \right)^2 (0.1)^{\frac{4}{3}} \right] \right\}$$
 (7)

With this calculation, Y_{OBS} is an additional parameter in the Holloman Spring 1999 thermosonde campaign model development data set and the Holloman Spring 1998 verification data set.

The Dewan optical turbulence parameterization was developed to convert standard radiosonde data into vertical profiles of C_n² at 300 m vertical resolution. To compare modified-Dewan optical turbulence parameterizations developed during this study with the Dewan parameterization, each high resolution profile used for model development and verification was smoothed to 300 m vertical resolution. Thus, the

development (20 profiles) and verification (8 profiles) data sets are composed of continuous data from 1.572 km to 29.772 km at 300 m vertical resolution, including parameters C_n^2 , calculated from the C_T^2 thermosonde measurement profiles; meteorological data including temperature, pressure, relative humidity, wind speed and wind direction from the radiosonde profiles; and the derived parameters temperature lapse rate, wind shear and Y_{OBS} .

Four model development and four model verification data sets were generated. Each data set includes the optical turbulence and meteorological data at 300 meter vertical resolution. Two data sets, development and verification, were designed to develop modified-Dewan parameterizations for use in the lower troposphere. These data heights range from 1.572 km to 5.472 km, intentionally avoiding the boundary layer. The Dewan parameterization consists of distinct algorithms for the troposphere and stratosphere. Thus, distinct development and verification data sets were generated for the troposphere and the stratosphere. The final development and verification data sets provide optical turbulence and meteorological data at 300 m vertical resolution for the combined troposphere/stratosphere from 1.572 km to 29.772 km.

For the troposphere development data set, heights range from 1.572 km to the tropopause, while the stratosphere development data set heights range from the 300 m level above the tropopause to 29.772 km. For the verification data set, the troposphere data set heights range from 1.572 km to 12.972 km, while the stratosphere data set heights range from 14.172 km to 27.672 km. It was necessary to define common height ranges for each profile of the troposphere and stratosphere verification data sets for the analysis of observed and parameterization-generated vertically integrated C_n^2 profiles. The stratosphere verification data set consists of six profiles because there were data

gaps in two profiles that precluded their use in calculating the vertically integrated ${\rm C_n}^2$ values. The tropopause height is determined by the World Meteorological Organization definition, which defines the tropopause height as the lowest height in the atmosphere where the lapse rate decreases to an average of 2°C/km for a 2 km layer.

4 MODIFIED-DEWAN OPTICAL TURBULENCE PARAMETERIZATIONS

The statistical relationships describing Dewan's Y parameter as a linear function of wind shear, shown in Equation (3), which form the basis of the Dewan parameterization, are often not found in atmospheric measurement data, thus this is an area to explore in developing modified-Dewan parameterizations to improve the reliability of C_n² profiles. Recent attempts, using thermosonde campaign data from different locations and seasons, to derive modified-Dewan parameterizations have concentrated on developing improved statistical relationships for Dewan's Y parameter as linear functions of wind shear, or as a combined linear function of wind shear and temperature lapse rate. These modified-Dewan parameterizations are sometimes successful in reducing systematic bias in C_n² profiles, but their overall performance did not significantly improve upon the Dewan parameterization. In this study, several modified-Dewan parameterizations are developed for the lower troposphere, the troposphere, the stratosphere, and the combined troposphere and stratosphere, creating new statistical relationships for Dewan's Y parameter as linear functions of wind shear, combined linear functions of wind shear and temperature lapse rate, fourth order polynomial functions of temperature lapse rate, and as combined linear functions of wind shear and fourth order polynomial functions of temperature lapse rate.

The modified-Dewan optical turbulence parameterizations described in this study were developed and tested using the Air Force Research Laboratory (AFRL) Holloman Spring 1998 and Holloman Spring 1999 thermosonde campaigns, that is, the parameterizations were developed using data from one season, Spring, and for one location, Holloman, New Mexico. The performance of the parameterizations in other seasons or locations is untested.

4.1 Model Development

Figure 1 shows the distribution of Y_{OBS}, as calculated from observations using Equation (7), and wind shear from the Holloman Spring 1999 troposphere development data set, and the linear relationship for Dewan's troposphere Y parameter described by

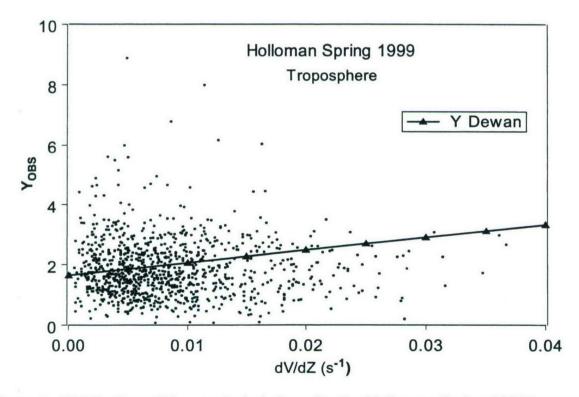


Figure 1. Distribution of Y_{OBS} and wind shear for the Holloman Spring 1999 troposphere development data set and Dewan's troposphere Y parameter plotted as a linear function of wind shear.

Equation (3a). The scatter plot suggests there is little relationship between Y_{OBS} and wind shear, with a wide range of Y_{OBS} observed for a particular value of wind shear. The correlation coefficient for the Y_{OBS} and wind shear observations is -0.02, also suggesting minimal relationship between the parameters. The line representing the Dewan troposphere Y parameter increases linearly as wind shear increases. This discrepancy has been found in several thermosonde campaign data sets from different locations and seasons and has led to a search for other parameters or combinations of parameters that affect the distribution of C_n^2 that may provide better statistical relationships with Y_{OBS} .

Figure 2 shows the distribution of Y_{OBS} and temperature lapse rate from the

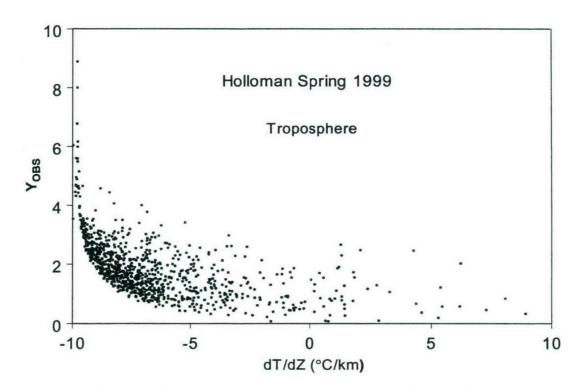


Figure 2. Distribution of Y_{OBS} and temperature lapse rate for the Holloman Spring 1999 troposphere development data set.

Holloman Spring 1999 troposphere development data set. The scatter plot suggests a nonlinear relationship with decreasing values of Y_{OBS} as lapse rate increases. The correlation coefficient for the Y_{OBS} and lapse rate observations is -0.54, suggesting considerably more relationship than is present in the Y_{OBS} and wind shear observations.

A distribution of observations similar to those displayed in Figure 1 and Figure 2 for the troposphere is observed in the lower troposphere (1.572 km to 5.472 km) and in the stratosphere model development data sets.

Numerous regression fits describing Y_{OBS} as a function of temperature lapse rate were explored for the observations displayed in Figure 2. A fourth order polynomial was determined to be the best regression fit describing the seemingly nonlinear relationship present in the Y_{OBS} and temperature lapse rate observations. Nonlinear regression fits were also attempted relating Y_{OBS} and the inverse of the temperature lapse rate, (dT/dZ)⁻¹, but it was difficult to fit the steep slope observed in the data. A linear regression fit describing Y_{OBS} as a function of temperature lapse rate was also developed to describe the tendency for Y_{OBS} to decrease with increasing temperature lapse rate.

Although there certainly appears to be a poor statistical relationship between Y_{OBS} and wind shear as shown in Figure 1, a linear regression fit describing Y_{OBS} as a function of wind shear was developed to test a modified-Dewan parameterization based on Dewan's original parameterization. Similar modified-Dewan parameterizations developed with other thermosonde campaign data sets have provided some statistical improvement over the Dewan model, but overall results were inconclusive and undocumented.

Two multi-parameter statistical relationships were developed describing Y_{OBS} as a function of both wind shear and temperature lapse rate: Y_{OBS} as a linear function of wind shear and a linear function of temperature lapse rate, and Y_{OBS} as a linear function of wind shear and a fourth order polynomial function of temperature lapse rate.

In summary, five modified-Dewan parameterizations were developed from the Holloman Spring 1999 development data set relating Y_{OBS} to wind shear, temperature lapse rate or a combination of the two parameters for the lower troposphere (1.572 km to 5.472 km), the troposphere, and the stratosphere. The Y parameter for the modified Dewan parameterizations is defined as Y*. The five modified-Dewan Y* parameterizations are:

$$Y^* = f(linear dV/dZ)$$
 (8a)

$$Y^* = f(linear dT/dZ)$$
 (8b)

$$Y^* = f(linear dV/dZ, linear dT/dZ)$$
 (8c)

$$Y^* = f(fourth order polynomial dT/dZ)$$
 (8d)

$$Y^*= f(linear dV/dZ, fourth order polynomial dT/dZ)$$
 (8e)

where dV/dZ represents wind shear and dT/dZ represents the temperature lapse rate. A combined troposphere/stratosphere modified-Dewan parameterization described by Equation (8d) was also developed using the entire Holloman Spring 1999 development data set. This parameterization was developed to determine if, unlike the Dewan parameterization which has distinct Y equations for the troposphere and stratosphere, one equation could adequately parameterize the entire atmosphere.

The Y* parameter is calculated at 300 m vertical resolution for each modified-Dewan parameterization described in Equation (8) and included in the Holloman Spring 1998 verification data sets where it can be directly compared to Y_{OBS} and Y calculated from the Dewan model. C_n² is calculated as a function of the modified-Dewan Y* parameterizations at 300 m vertical resolution by modifying Equation (2) to

$$C_n^2 = 2.8 \left(\frac{\left(79x10^{-6}P\right)}{T^2} \right)^2 (0.1)^{4/3} \left(\frac{\partial T}{\partial Z} + \gamma_d \right)^2 10^{\gamma *}$$
 (9)

where Y* replaces Y. These values of C_n^2 calculated as a function of Y* are included in the verification data sets and can be directly compared to the C_n^2 observations and C_n^2 values calculated from the AFRL Dewan and CLEAR1 optical turbulence models.

4.2 Lower Troposphere Parameterizations

Figure 3 shows the distribution of YOBS and wind shear from the Holloman Spring

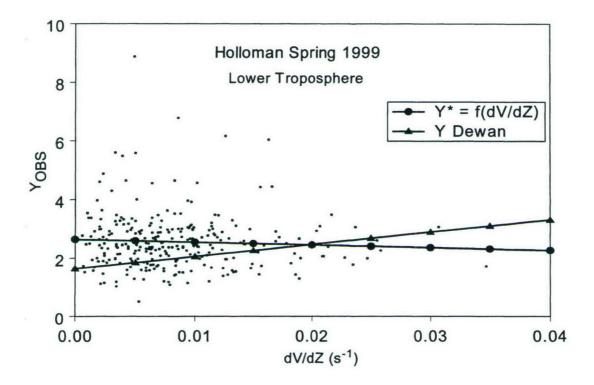


Figure 3. Distribution of Y_{OBS} and wind shear for the Holloman Spring 1999 lower troposphere development data set, the regression fit for Y^* as a linear function of wind shear, and Dewan's troposphere Y parameter plotted as a linear function of wind shear.

1999 lower troposphere (1.572 km to 5.472 km) development data set, the regression fit for Y* as a linear function of wind shear, and the linear relationship for Dewan's troposphere Y parameter. The scatter plot suggests there is little relationship between Y_{OBS} and wind shear. The plot for the Y* parameterization as a linear function of wind shear (Equation (10a)) shows a slight decrease in Y* as wind shear increases, whereas the Dewan troposphere model shows a steady increase in Y as wind shear increases. The correlation coefficient for the Y_{OBS} and wind shear observations is -0.05.

Figure 4 shows the distribution of Y_{OBS} and temperature lapse rate from the Holloman Spring 1999 lower troposphere development data set. The scatter plot

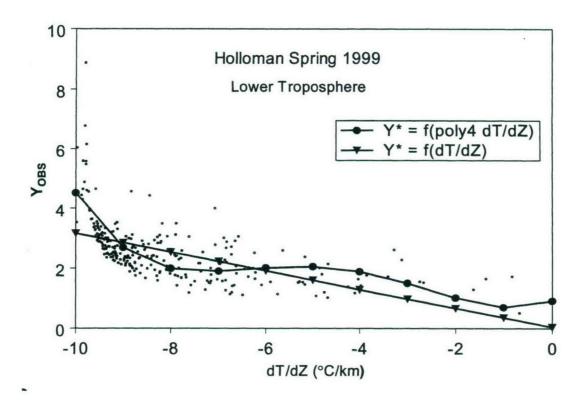


Figure 4. Distribution of Y_{OBS} and temperature lapse rate for the Holloman Spring 1999 lower troposphere development data set, the regression fit for Y^* as a fourth order polynomial of temperature lapse rate, and Y^* as a linear function of temperature lapse rate.

suggests a nonlinear relationship with decreasing values of Y_{OBS} as lapse rate increases. The correlation coefficient for the Y_{OBS} and lapse rate observations is -0.56, significantly greater than the correlation coefficient for the Y_{OBS} and wind shear observations.

The plot for the Y* parameterization as a fourth order polynomial function of temperature lapse rate (Equation (10d)) is a reasonable fit. At the lowest temperature lapse rates, near -9.8°C/km, the regression fit does not capture the Y_{OBS} values greater than about 4.5, though these observations are relatively rare. The curve captures the significant decrease of Y_{OBS} from about -9.8°C/km to -8°C/km and the gradual decrease of Y_{OBS} from about -8°C/km to 0°C/km. The plot for the Y* parameterization as a linear function of temperature lapse rate (Equation (10b)) captures the gradual decrease of Y_{OBS} with increasing temperature lapse rate, though it misses the higher values of Y_{OBS} near -9°C/km and results in unrealistically low values of Y_{OBS} near 0°C/km. Note that most of the observations for this development data set in the lower troposphere are found from about -9.8°C/km to -5°C/km where both curves provide reasonable fits, though the fourth order polynomial fit is certainly superior.

$$Y^* = 2.6470 - 9.2651 \times dV/dZ$$
 (10a)

$$Y^* = 0.0427 - 0.3117 \times dT/dZ$$
 (10b)

$$Y^* = -0.2007 + 16.2001 \times dV/dZ - 0.3251 \times dT/dZ$$
 (10c)

 $Y^* = 0.9229 + 0.6565 \times dT/dZ + 0.5255 \times (dT/dZ)^2 + 0.0972 \times (dT/dZ)^3$

$$+0.0055 \times (dT/dZ)^4$$
 (10d)

$$Y^* = 2.9767 + 27.9804 \times dV/dZ + 2.9012 \times dT/dZ + 1.1843 \times (dT/dZ)^2$$

$$+ 0.1741 \times (dT/dZ)^3 + 0.0086 \times (dT/dZ)^4$$
 (10e)

The Y* parameterization as a linear function of wind shear and a linear function of temperature lapse rate is shown in Equation (10c). The Y* parameterization as a linear function of wind shear and a fourth order polynomial of temperature lapse rate is shown in Equation (10e).

4.3 Troposphere and Stratosphere Parameterizations

4.3.1 Troposphere Parameterizations

Figure 5 shows the distribution of Y_{OBS} and wind shear from the Holloman Spring 1999 troposphere development data set, the regression fit for Y* as a linear function of wind shear, and the linear relationship for Dewan's troposphere Y parameter. The

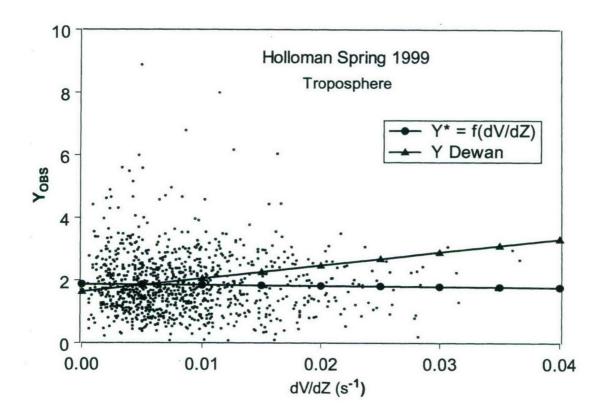


Figure 5. Distribution of Y_{OBS} and wind shear for the Holloman Spring 1999 troposphere development data set, the regression fit for Y^* as a linear function of wind shear, and Dewan's troposphere Y parameter plotted as a linear function of wind shear.

scatter plot suggests there is little relationship between Y_{OBS} and wind shear. The plot for the Y* parameterization as a linear function of wind shear (Equation (11a)) shows a very slight decrease in Y* as wind shear increases, whereas the Dewan troposphere model shows a steady increase in Y as wind shear increases. The correlation coefficient for the Y_{OBS} and wind shear observations is -0.02.

Figure 6 shows the distribution of Y_{OBS} and temperature lapse rate from the Holloman Spring 1999 troposphere development data set. The scatter plot suggests a nonlinear relationship with decreasing values of Y_{OBS} as temperature lapse rate increases. The correlation coefficient for the Y_{OBS} and temperature lapse rate observations is -0.54.

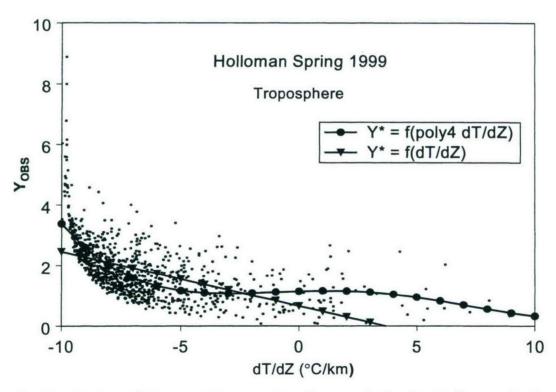


Figure 6. Distribution of Y_{OBS} and temperature lapse rate for the Holloman Spring 1999 troposphere development data set, the regression fit for Y^* as a fourth order polynomial of temperature lapse rate, and Y^* as a linear function of temperature lapse rate.

The plot for the Y* parameterization as a fourth order polynomial function of temperature lapse rate (Equation (11d)) is a reasonable fit. As in Figure 4 for the lower troposphere, at the lowest temperature lapse rates, near -9.8°C/km, the regression fit does not capture the Y_{OBS} values greater than about 3.5, though these observations are relatively rare. The curve captures the significant decrease of Y_{OBS} from about -9.8°C/km to -5°C/km and the gradual decrease of Y_{OBS} from about -5°C/km to 10°C/km. The plot for the Y* parameterization as a linear function of temperature lapse rate (Equation (11b)) captures the gradual decrease of Y_{OBS} with increasing temperature lapse rate, though it misses the higher values of Y_{OBS} near -9°C/km and results in unrealistically low values of Y_{OBS} at temperature lapse rates greater than 0°C/km. Note that most of the observations for this development data set in the troposphere are found from about -9.8°C/km to 0°C/km where both curves provide reasonable fits, although as in the lower troposphere, the fourth order polynomial fit is certainly superior.

$$Y^* = 1.8754 - 2.5398 \times dV/dZ$$
 (11a)

$$Y^* = 0.6567 - 0.1791 \times dT/dZ$$
 (11b)

$$Y^* = 0.2707 + 29.0870 \times dV/dZ - 0.1996 \times dT/dZ$$
 (11c)

$$Y^* = 1.1408 + 0.0226 \times dT/dZ - 0.0070 \times (dT/dZ)^2 - 0.0017 \times (dT/dZ)^3$$

$$+ 0.0001 \times (dT/dZ)^4$$
 (11d)

$$Y^* = 0.7152 + 30.6024 \times dV/dZ + 0.0003 \times dT/dZ - 0.0057 \times (dT/dZ)^2$$

$$-0.0016 \times (dT/dZ)^3 + 0.0001 \times (dT/dZ)^4$$
 (11e)

The Y* parameterization as a linear function of wind shear and a linear function of temperature lapse rate is shown in Equation (11c). The Y* parameterization as a

linear function of wind shear and a fourth order polynomial of temperature lapse rate is shown in Equation (11e).

4.3.2 Stratosphere Parameterizations

Figure 7 shows the distribution of Y_{OBS} and wind shear from the Holloman Spring 1999 stratosphere development data set, the regression fit for Y* as a linear function of wind shear, and the linear relationship for Dewan's stratosphere Y parameter. Unlike the lower troposphere plot shown in Figure 3 and the troposphere plot shown in Figure 5 where most Y_{OBS} values are between 0 and 4, most Y_{OBS} values are in a narrower range, between 0 and 2. As in Figure 3 and Figure 5 for the lower troposphere and troposphere, the scatter plot suggests there is little relationship between Y_{OBS} and wind

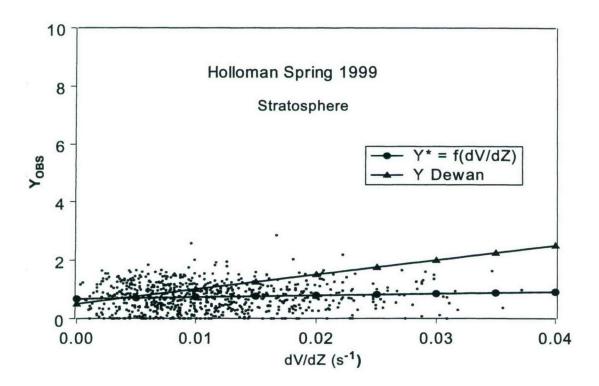


Figure 7. Distribution of Y_{OBS} and wind shear for the Holloman Spring 1999 stratosphere development data set, the regression fit for Y* as a linear function of wind shear, and Dewan's stratosphere Y parameter plotted as a linear function of wind shear.

shear. The plot for the Y* parameterization as a linear function of wind shear (Equation (12a)) shows a very slight increase in Y* as wind shear increases, whereas the Dewan stratosphere model shows a significantly larger increase in Y as wind shear increases. The correlation coefficient for the Y_{OBS} and wind shear observations is 0.09.

Figure 8 shows the distribution of Y_{OBS} and temperature lapse rate from the Holloman Spring 1999 stratosphere development data set. The scatter plot is not nearly as definitive in suggesting the nonlinear relationship for decreasing values of Y_{OBS} as lapse rate increases as is clearly shown in the scatter plots for the lower troposphere and the troposphere. However, there is a tendency for larger values of Y_{OBS} for lapse rates less than -5°C/km, though there are few data points. The correlation coefficient for the Y_{OBS} and lapse rate observations is -0.54.

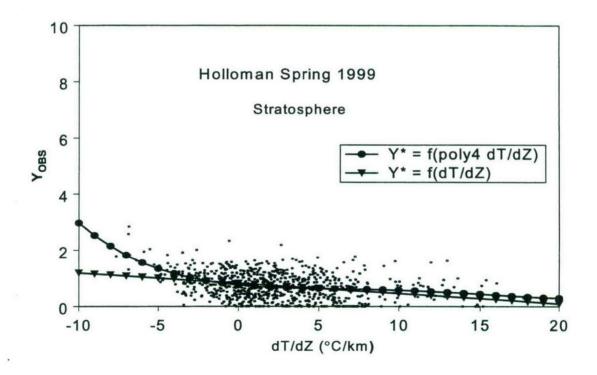


Figure 8. Distribution of Y_{OBS} and temperature lapse rate for the Holloman Spring 1999 stratosphere development data set, the regression fit for Y^* as a fourth order polynomial of temperature lapse rate, and Y^* as a linear function of temperature lapse rate.

The plots for the Y* parameterizations as a fourth order polynomial (Equation (12d)) and a linear (Equation (12b)) function of lapse rate are nearly identical for temperature lapse rates greater than -3°C/km showing a gradual decrease of Y_{OBS} with increasing temperature lapse rate.

$$Y^* = .6712 + 5.8789 \times dV/dZ$$
 (12a)

$$Y^* = .8217 - .0365 \times dT/dZ$$
 (12b)

$$Y^* = .7200 + 9.5646 \times dV/dZ - .0392 \times dT/dZ$$
 (12c)

$$Y^* = .7628 - .0541 \times dT/dZ + .0086 \times (dT/dZ)^2 - .0007 \times (dT/dZ)^3$$
$$+ .00002 \times (dT/dZ)^4$$
(12d)

$$Y^* = .6763 + 8.1569 \times dV/dZ - .0536 \times dT/dZ + .0084 \times (dT/dZ)^2$$
$$- .0007 \times (dT/dZ)^3 + .00002 \times (dT/dZ)^4$$
(12e)

The Y* parameterization as a linear function of wind shear and a linear function of temperature lapse rate is shown in Equation (12c). The Y* parameterization as a linear function of wind shear and a fourth order polynomial of temperature lapse rate is shown in Equation (12e).

4.3.3 Troposphere/Stratosphere Parameterization

Examination of the regression fits for the parameterizations for Y* as a fourth order polynomial function of temperature lapse rate for the lower troposphere, the troposphere and the stratosphere, shown in Figure 4, Figure 6, and Figure 8 respectively, suggests that a single fourth order polynomial regression fit may adequately represent the entire atmosphere. For application purposes, it is certainly advantageous if one parameterization could adequately model the entire atmosphere. Thus a

parameterization describing Y* as a fourth order polynomial of temperature lapse rate was developed using the Holloman Spring 1999 combined troposphere/stratosphere development data set.

Figure 9 shows the distribution of Y_{OBS} and temperature lapse rate from the Holloman Spring 1999 combined troposphere/stratosphere development data set. The scatter plot suggests a nonlinear relationship with decreasing values of Y_{OBS} as temperature lapse rate increases. The correlation coefficient for the Y_{OBS} and temperature lapse rate observations is -0.66. The plot for the Y^* parameterization as a fourth order polynomial function of temperature lapse rate (Equation (13))

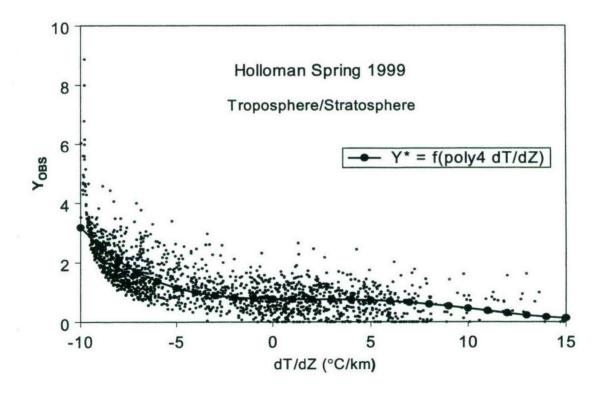


Figure 9. Distribution of Y_{OBS} and temperature lapse rate for the Holloman Spring 1999 combined troposphere/stratosphere development data set and the regression fit for Y* as a fourth order polynomial of temperature lapse rate.

$$Y^* = 0.7716 - 0.0104 \times dT/dZ + 0.0056 \times (dT/dZ)^2 - 0.0013 \times (dT/dZ)^3 + 0.00005 \times (dT/dZ)^4$$
(13)

is a reasonable fit. At the lowest temperature lapse rates, near -9.8°C/km, the regression fit does not capture the Y_{OBS} values greater than about 3.2, though these observations are relatively rare. The curve captures the significant decrease of Y_{OBS} from about -9.8°C/km to -3°C/km and the gradual decrease of Y_{OBS} from about -3°C/km to 15°C/km.

5 RESULTS

Each of the Y* parameterizations developed from the Holloman Spring 1999 development data sets are evaluated using the Holloman Spring 1998 verification data sets. The Y* parameterizations are compared to observations and the Dewan and the CLEAR1 parameterizations. Each parameterization's Y* values are compared to the Y_{OBS} values calculated from the thermosonde observations and the Dewan parameterization Y values. The C_n^2 values calculated for each Y* parameterization are compared to the measured thermosonde C_n^2 observations and the Dewan and CLEAR1 parameterizations' C_n^2 values. Each Y* parameterization C_n^2 profile is vertically integrated and compared to the vertically integrated C_n^2 values derived from observations and the Dewan and CLEAR1 parameterizations.

The five Y* parameterizations described in Equations (8a) through (8e) are evaluated for each of the lower troposphere, the troposphere, and the stratosphere in the tables below. The Y* parameterizations are labeled in the tables as y*shear: Y* as a linear function of wind shear (Equations (10a), (11a), (12a)), y*dtdz: Y* as a linear function of temperature lapse rate (Equations (10b), (11b), (12b)), y*sheardtdz: Y* as a

linear function of wind shear and a linear function of temperature lapse rate (Equations (10c), (11c), (12c)), y*dtdzpoly4: Y* as a fourth order polynomial function of temperature lapse rate (Equation (10d), (11d), (12d)) and y*sheardtdzpoly4: Y* as a linear function of wind shear and a fourth order polynomial function of temperature lapse rate (Equations (10e), (11e), (12e)). The parameterization y*tropstratdtdzpoly4 describes Y* as a fourth order polynomial function of lapse rate for the combined troposphere/stratosphere (Equation (13)) and is compared to the lower troposphere, troposphere and stratosphere parameterizations.

5.1 Lower Troposphere Parameterizations

The five lower troposphere Y* parameterizations (Equations (10a) – (10e)) shown in Table 1 perform better at diagnosing Y_{OBS} than the Dewan parameterization for each statistical measure presented. The Dewan parameterization exhibits a significant negative bias of -0.66 while the Y* parameterizations exhibit minimal bias. The Dewan parameterization Y parameter correlates very poorly with the observed Y parameter, with a correlation coefficient of 0.11. The y*dtdzpoly4 and y*sheardtdzpoly4 parameterizations perform the best with the smallest bias and root mean square error (rms) and the largest correlation with the observed Y parameter, 0.70 and 0.75

Table 1. Mean, Bias, Root Mean Square Error, and Correlation Coefficient for the Dewan troposphere Y parameter and the lower troposphere Y* parameterizations using the lower troposphere verification data set.

Statistics	yobs	ydewan	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4
Mean	2.62	1.97	2.58	2.68	2.68	2.60	2.58
BIAS		-0.66	-0.05	0.06	0.06	-0.03	-0.04
RMS		1.07	0.86	0.72	0.70	0.61	0.57
Correlation		0.11	-0.11	0.57	0.61	0.70	0.75

respectively. There is no Y parameter in the CLEAR1 model, thus it is not evaluated in the Y parameter comparison tables.

Figure 10 shows the distribution of Y_{OBS} plotted against the Y parameter calculated for the y*sheardtdzpoly4, Dewan, y*sheardtdz and y*shear parameterizations. The distribution of Y* values for the y*sheardtdzpoly4 parameterization (Figure 10A) is superior to the other parameterizations, with most data points distributed along the diagonal line as desired. The distribution suggests a strong correlation between the y*sheardtdzpoly4 Y parameter and Y_{OBS} as shown in Table 1. The y*sheardtdzpoly4 parameterization has a noticeable negative bias for values of

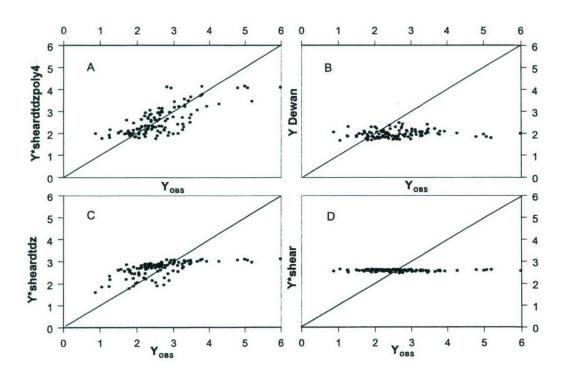


Figure 10. Distribution of Y_{OBS} and Y from the A. Y*sheardtdzpoly4, B. Dewan, C. Y*sheardtdz and D. Y*shear lower troposphere parameterizations for the lower troposphere verification data set.

 Y_{OBS} greater than about 4. The Y parameter values for the Dewan parameterization (Figure 10B) range from about 1.5 to 2.5 while Y_{OBS} varies from about 1.0 to 5.0, exhibiting the large negative bias shown in Table 1. The distribution suggests little correlation between the Dewan Y parameter and Y_{OBS} as shown in Table 1. The Y* values for the y*sheardtdz parameterization (Figure 10C) shows a tendency to follow the diagonal line for Y_{OBS} values from 1.0 to 3.0, but fails to diagnose Y_{OBS} values greater than about 3.5, although it greatly improves upon the negative bias exhibited by the Dewan parameterization. The y*shear parameterization (Figure 10D) which is similar to the Dewan parameterization (both Y values are linear functions of wind shear only) shows very little variation with a mean value of 2.58, though the distribution about the diagonal line suggests a significant improvement in the bias compared to the Dewan Y parameterization which has a mean value of 1.97.

The plot for y*dtdzpoly4 is very similar to the y*sheardtdzpoly4 plot and the y*dtdz plot is very similar to the y*sheardtdz plot, both exhibiting only slightly more spread about the diagonal as is suggested by the statistics in Table 1, thus these plots are not shown.

The five lower troposphere Y* parameterizations shown in Table 2 perform better than the Dewan and CLEAR1 parameterizations at diagnosing the mean $\log C_n^2$ values for each parameterization. The Y* parameterizations all have significantly lower bias and rms than the Dewan parameterization. Logically, the significant negative bias for the Dewan Y parameter shown in Table 1, results in a significant $\log C_n^2$ negative bias for the Dewan parameterization, which would result in under forecasting the intensity of optical turbulence in the lower troposphere. The CLEAR1 parameterization shows a modest positive bias for the $\log C_n^2$ values which would result in over forecasting the

Table 2. Mean, Bias, Root Mean Square Error, and Correlation Coefficient of $\log C_n^2$ for the Dewan, CLEAR1, and lower troposphere Y* parameterizations using the lower troposphere verification data set.

Statistics	Obs	dewan	clear1	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4
Mean	-16.67	-17.33	-16.28	-16.72	-16.61	-16.61	-16.70	-16.71
BIAS		-0.66	0.39	-0.05	0.06	0.06	-0.03	-0.04
RMS		1.07	0.67	0.86	0.72	0.70	0.61	0.57
Correlation		0.44	0.45	0.35	0.30	0.35	0.33	0.44

intensity of optical turbulence in the lower troposphere. Similar to the Y parameter statistics shown in Table1, the Y* parameterizations show little bias in diagnosing the log C_n^2 values in Table 2. The Dewan, CLEAR1 and y*sheardtdzpoly4 parameterizations exhibit the largest correlation with the log C_n^2 observations. Overall, the y*sheardtdzpoly4 and y*dtdzpoly4 parameterizations perform the best in diagnosing log C_n^2 .

Figure 11 demonstrates the benefit of accurately diagnosing the Y parameter. In Figure 11 A the distribution of the Dewan and $y^*sheardtdzpoly4$ Y parameter about the diagonal is very good. Figure 11 B shows that the C_n^2 values calculated from the Dewan and $y^*sheardtdzpoly4$ Y parameter values agree quite well with the observed C_n^2 values. The y^*shear parameterization Y parameter distribution in Figure 11 A shows a significant positive bias compared to the observed Y values. In Figure 11 B, the y^*shear C_n^2 profile shows a significant positive bias in C_n^2 values throughout the profile. In Figure 11C the $y^*sheardtdzpoly4$ Y parameter distribution is excellent, with all Y parameter values falling very close to the diagonal. The $y^*sheardtdzpoly4$ C_n^2 profile in Figure 11 D compares quite well to the observed C_n^2 profile. The Dewan

parameterization Y values in Figure 11C show a significant negative bias which is reflected in the C_n^2 profile in Figure 11 D.

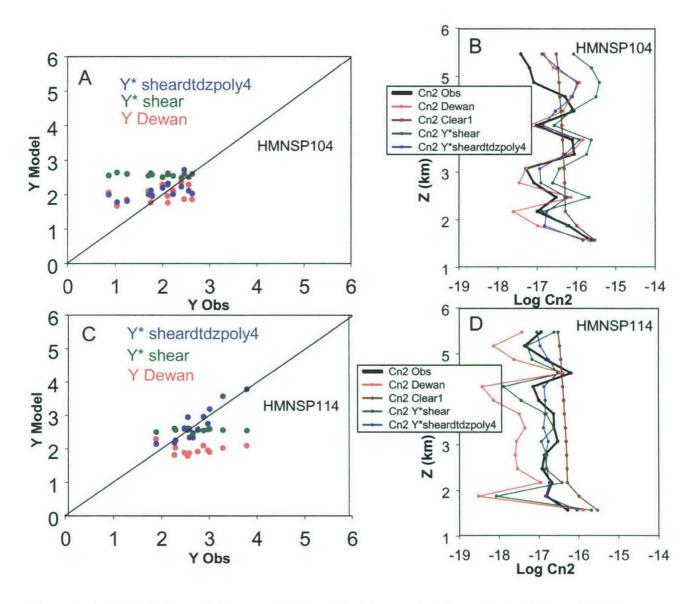


Figure 11. Distribution of Y_{OBS} and Y from the Dewan, Y*sheardtdzpoly4, and Y*shear lower troposphere parameterizations for the lower troposphere verification data set for thermosonde flights A. hmnsp104 and C. hmnsp114. C_n^2 profiles for the Dewan, CLEAR1, Y*sheardtdzpoly4, and Y*shear lower troposphere Y* parameterizations for thermosonde flights B. hmnsp104 and D. hmnsp114.

The Y* parameterizations developed for the troposphere (Equations (11a) – (11e)) were also tested using the lower troposphere verification data set. This test was conducted to determine if a parameterization developed specifically for the lower troposphere (1.572 km to 5.472 km) performed better in the lower troposphere than parameterizations developed for the entire troposphere. The *y*tropstratdtdzpoly4* parameterization (Equation (13)) developed as a single parameterization for the combined troposphere/stratosphere was also tested using the lower troposphere verification data set. This test was conducted to determine if a parameterization developed for the lower troposphere performed better in the lower troposphere than a parameterization developed for the combined troposphere/stratosphere. The statistics for these parameterizations are displayed in Table 3.

Comparing the statistics listed in Table 1 and Table 3, it is obvious that the Y* parameterizations developed specifically for the lower troposphere perform better at diagnosing the Y parameter in the lower troposphere than Y* parameterizations developed for the entire troposphere or the combined troposphere/stratosphere. For each Y* parameterization in Table 3, the means are undesirably smaller, the bias is larger and the rms is larger than those in Table 1. Interestingly, except for the y*shear parameterization, each Y* parameterization performs better than the Dewan parameterization for each statistical measure. The y*dtdzpoly4 and y*sheardtdzpoly4 parameterizations developed for the troposphere perform quite well, significantly better than the Dewan parameterization. The y*tropstratdtdzpoly4 parameterization developed for the troposphere, but poorer than the y*dtdzpoly4 parameterization developed for the troposphere, but poorer than the y*dtdzpoly4 parameterization developed for the lower troposphere. Though several Y*

parameterizations developed for the entire troposphere perform reasonably well in the lower troposphere, Y* parameterizations developed for the lower troposphere perform better in the lower troposphere.

Table 3. Mean, Bias, Root Mean Square Error, and Correlation Coefficient for the Dewan troposphere Y parameter, the troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization using the lower troposphere verification data set.

Statistics	yobs	ydewan	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4	y*tropstratdtdzpoly4
Mean	2.62	1.97	1.86	2.17	2.19	2.34	2.36	2.34
BIAS		-0.66	-0.77	-0.45	-0.44	-0.28	-0.26	-0.28
RMS		1.07	1.15	0.88	0.85	0.72	0.68	0.73
Correlation		0.11	-0.11	0.57	0.63	0.63	0.68	0.61

Close examination of the plots shown in Figure 10 and Figure 12 shows the tendency for Y parameter values for the troposphere Y* parameterizations to be undesirably smaller than Y parameter values for the lower troposphere Y* parameterizations. This is most obvious in Figure 12 D where the y*shear values average around 1.9 whereas they average around 2.6 in Figure 10 D. These results suggest that Y* parameterizations developed for the troposphere or the combined troposphere/stratosphere will diagnose Y parameter values that are too small in the lower troposphere.

Comparing the statistics listed in Table 2 and Table 4, the Y* parameterizations developed specifically for the lower troposphere perform better at diagnosing log C_n² in the lower troposphere than the Y* parameterizations developed for the entire troposphere or the combined troposphere/stratosphere. For each Y* parameterization

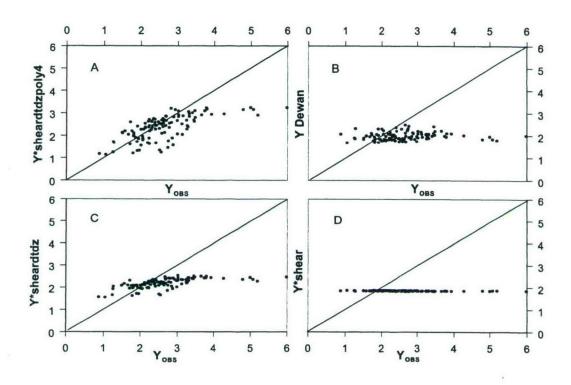


Figure 12. Distribution of Y_{OBS} and Y from the A. Y*sheardtdzpoly4, B. Dewan, C. Y*sheardtdz and D. Y*shear troposphere parameterizations for the lower troposphere verification data set.

in Table 4, the means are undesirably smaller, the bias is larger and the rms is larger than those is Table 2. Except for the y*shear parameterization, the Y* parameterizations perform better than the Dewan parameterization. The y*dtdzpoly4 and y*sheardtdzpoly4 parameterizations developed for the troposphere perform guite well in diagnosing log C_n². The y*tropstratdtdzpoly4 parameterization developed for the combined troposphere/stratosphere performs very similarly to the v*dtdzpolv4 parameterization developed for the troposphere, but poorer than the y*dtdzpoly4 parameterization developed for the lower troposphere. These results suggest that developed parameterizations for the troposphere or the combined troposphere/stratosphere will diagnose log C_n² values that are too small in the lower troposphere, thus underestimating the intensity of optical turbulence in the lower troposphere.

Table 4. Mean, Bias, Root Mean Square Error, and Correlation Coefficient of $\log {C_n}^2$ for the Dewan, CLEAR1, troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization using the lower troposphere verification data set.

Statistics	obs	dewan	clear1	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4	y*tropstratdtdzpoly4
Mean	-16.67	-17.33	-16.28	-17.44	-17.12	-17.11	-16.95	-16.93	-16.95
BIAS		-0.66	0.39	-0.77	-0.45	-0.44	-0.28	-0.26	-0.28
RMS		1.07	0.67	1.15	0.88	0.85	0.72	0.68	0.73
Correlation		0.44	0.45	0.36	0.34	0.40	0.24	0.34	0.26

In addition to evaluating the Y parameter and log C_n² for each parameterization. each thermosonde and parameterization C_n² profile was vertically integrated. For many optical propagation applications, vertically or horizontally integrated measures of C_n² are of primary importance. Table 5 shows the vertically integrated C_n² values for the eight lower troposphere thermosonde profiles, Figure 13 displays the results graphically. Nearly all the Y* parameterizations perform better than the Dewan parameterization. indicated with a "+" following the integrated C_n² value in Table 5. The y*sheardtdzpoly4 parameterization performs better than the Dewan parameterization for every profile. while several Y* parameterizations perform better than Dewan parameterizations for seven of the eight profiles. The Y* parameterizations also perform well compared to the CLEAR1 parameterization. The y*sheardtdzpoly4 parameterization performs better than the CLEAR1 parameterizations for six of the eight profiles, indicated with a "~" following the integrated C_n² value. The Dewan parameterization exhibits a significant negative bias in diagnosing the vertically integrated value of C_n², consistent with the negative bias in diagnosing Y and log C_n². The CLEAR1 parameterization produced the

smallest bias demonstrating its usefulness as a climatological model. The result must be considered somewhat fortuitous since the departure from the observed vertically integrated C_n^2 value tended to be evenly distributed, as shown in Figure 13. The y*sheardtdzpoly4 parameterization produced the smallest rms.

Figure 13 displays the comparison of vertically integrated C_n^2 profiles very well. The Dewan parameterization clearly demonstrates a negative bias. All parameterizations considerably underdiagnose the vertically integrated C_n^2 value for profile 2. The $y^*sheardtdzpoly4$ and $y^*dtdzpoly4$ parameterizations perform quite well in diagnosing the high vertically integrated C_n^2 value for profile 3. In general, Figure 13 suggests that except for profiles 2 and 3, most Y* parameterizations perform reasonably well in diagnosing the vertically integrated C_n^2 value in the lower troposphere.

Figure 14 displays the lower troposphere C_n^2 profiles for each observed thermosonde sounding and for each parameterization. There is a wide range of skill displayed in individual C_n^2 profiles. In Figure 14 A, the hmnsp104 Dewan and $y^*sheardtdzpoly4\ C_n^2$ profiles perform very well. There are very good feature matches, C_n^2 maximums and minimums of similar magnitude at the same altitude, with the observed C_n^2 values throughout the profiles. In Figure 14 G, the hmnsp114 $y^*sheardtdzpoly4\ C_n^2$ profile performs similarly well, while the Dewan parameterization underestimates C_n^2 through much of the profile. In Figure 14 E, the hmnsp112 profiles, all parameterizations perform relatively poorly. The Dewan parameterization tends to underdiagnose C_n^2 while the Y* parameterizations tend to overdiagnose C_n^2 throughout each profile. In most profiles, the Dewan parameterization has a strong

parameterizations, and the combined troposphere/stratosphere Y* parameterization using the lower troposphere verification data set. Bias, Root Mean Square Error, and Correlation Coefficient of the vertically integrated Cn² profiles for each parameterization are shown. "+" indicates the parameterization performed better than the Dewan parameterization, "~" indicates the parameterization performed better than the CLEAR1 parameterization, "#" indicates the rank of the Table 5. Vertically integrated C_n² values for eight thermosonde profiles for the Dewan, CLEAR1, lower troposphere Y* parameterization for that profile.

		The second secon	The second secon	The second secon	The second secon	The second secon			
	ops	dewan	clear1	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4	y*sheardtdz y*dtdzpoly4 y*sheardtdzpoly4 y*tropstratdtdzpoly4
hmnsp104	1.67E-13	hmnsp104 1.67E-13 2.09E-13 ~3 2.25E-13 5	2.25E-13 5	5.97E-13 7	2.24E-13 ~4	2.27E-13 6	1.79E-13+~1	5.97E-13 7 2.24E-13 ~4 2.27E-13 6 1.79E-13+~1 1.89E-13+~2	7.43E-14 8
hmnsp107	5.10E-13	1.19E-13 7	2.25E-13+3	3.97E-13+~1	2.21E-13+4	2.28E-13+~2	1.23E-13+6	hmnsp107 5.10E-13 1.19E-13 7 2.25E-13+3 3.97E-13+~1 2.21E-13+4 2.28E-13+~2 1.23E-13+6 1.29E-13+5	7.64E-14 8
hmnsp108	4.78E-13	5.28E-14 8	2.25E-13+ 4	2.54E-13+~3	1.71E-13+5	1.62E-13+6	3.22E-13+~2	hmnsp108 4.78E-13 5.28E-14 8 2.25E-13+ 4 2.54E-13+~3 1.71E-13+ 5 1.62E-13+ 6 3.22E-13+~2 3.52E-13+~1	7.56E-14+7
hmnsp111	1.63E-13	hmnsp111 1.63E-13 2.85E-14 8 2.25E-13+	2.25E-13+ 1	8.15E-14+4	8.15E-14+4 8.84E-14+2 8.44E-14+3 6.06E-14+5	8.44E-14+3	6.06E-14+ 5	5.77E-14+ 6	3.90E-14+7
hmnsp112	8.13E-14	hmnsp112 8.13E-14 2.78E-14 ~6 2.25E-13 8	2.25E-13 8	1.39E-13 ~7	1.39E-13 ~7 1.25E-13+~5 1.15E-13+~4 7.79E-14+~1	1.15E-13+~4	7.79E-14+~1	6.84E-14+~2	5.31E-14+~3
hmnsp113	1.07E-13	hmnsp113 1.07E-13 1.69E-14 ~7 2.25E-13 8	2.25E-13 8	7.36E-14+~3	1.02E-13+~1	9.99E-14+~2	6.15E-14+~4	7.36E-14+~3 1.02E-13+~1 9.99E-14+~2 6.15E-14+~4 5.82E-14+~5	4.63E-14+~6
hmnsp114	8.19E-14	hmnsp114 8.19E-14 4.25E-14 ~7 2.25E-13 8	2.25E-13 8	8.90E-14+~1	1.06E-13+~4	1.16E-13+~5	6.81E-14+~3	8.90E-14+~1 1.06E-13+~4 1.16E-13+~5 6.81E-14+~3 7.41E-14+~2	4.71E-14+~6
hmnsp115	1.69E-13	hmnsp115 1.69E-13 1.18E-13 ~4 2.25E-13 5		2.42E-13 7	2.42E-13 7 1.67E-13+~1 1.88E-13+~2 9.64E-14 6	1.88E-13+~2	9.64E-14 6	1.20E-13+~3	6.04E-14 8
BIAS		-1.43E-13 7 5.35E-15+	~	1.44E-14+2	-6.93E-14+ 4	-6.72E-14+3	-9.61E-14+ 6	1.44E-14+ 2 -6.93E-14+ 4 -6.72E-14+ 3 -9.61E-14+ 6 -8.86E-14+ 5	-1.61E-13 8
RMS		2.15E-13 6	1.62E-13+ 4	1.82E-13+ 5	1.54E-13+~2	1.55E-13+~3	1.55E-13+~3	2.15E-13 6 1.62E-13+4 1.82E-13+5 1.54E-13+~2 1.55E-13+~3 1.55E-13+~3 1.49E-13+~1	2.21E-13 7
Correlation	1.00	0.22	00.00	0.41 +	0.61 +	0.56 +	0.68 +	0.68 +	0.75 +

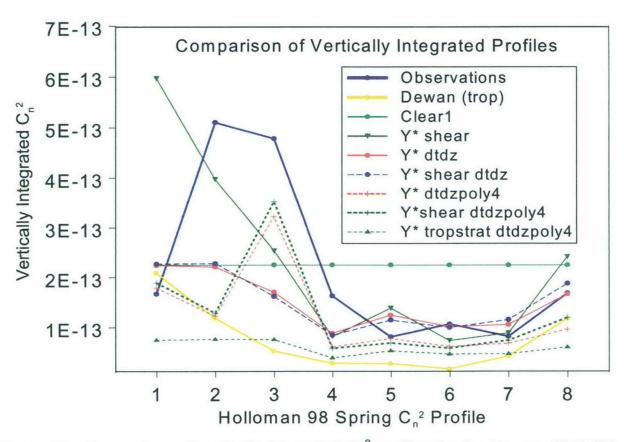


Figure 13. Comparison of vertically integrated C_n^2 profiles for the Dewan, CLEAR1, lower troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization using the lower troposphere verification data set. The numbers 1 thru 8 on the x-axis correspond to the thermosonde flights hmnsp104 thru hmnsp115 displayed in Table 5.

tendency to underestimate C_n^2 . The $y^*sheardtdzpoly4$ parameterization has a tendency to underestimate C_n^2 maximums and minimums, though it follows the observed profile throughout the lower troposphere closer than the other parameterizations. The $y^*tropstratdtdzpoly4$ parameterization, developed to test the usefulness of a single parameterization for the combined troposphere/stratosphere, performs well statistically in diagnosing the Y parameter and log C_n^2 in the lower troposphere, but its profiles often lack any variation with height as shown in Figure 14 A and Figure 14 B.

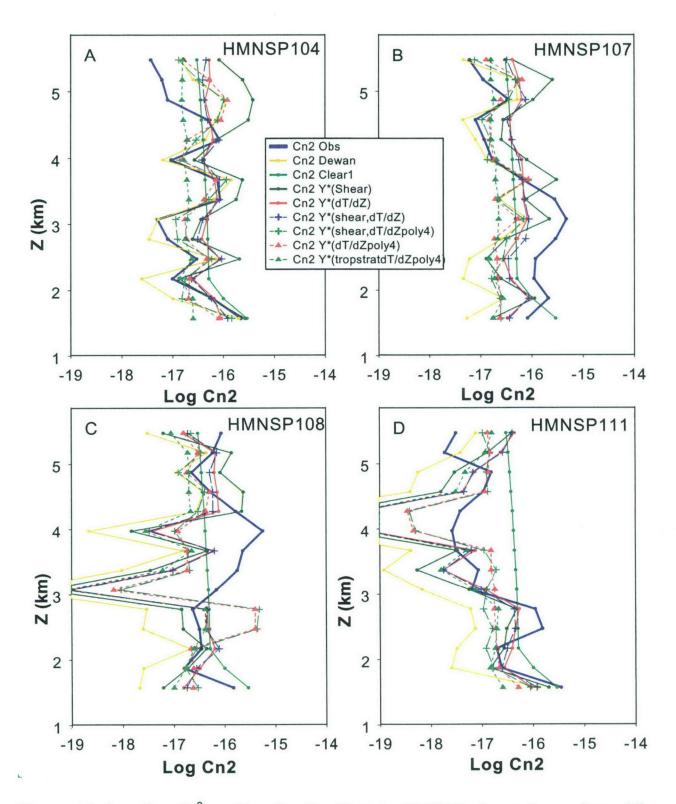


Figure 14 A - D. ${\rm C_n}^2$ profiles for the Dewan, CLEAR1, lower troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization, using the lower troposphere verification data set for thermosonde flights A. hmnsp104, B. hmnsp107, C. hmnsp108 and D. hmnsp111.

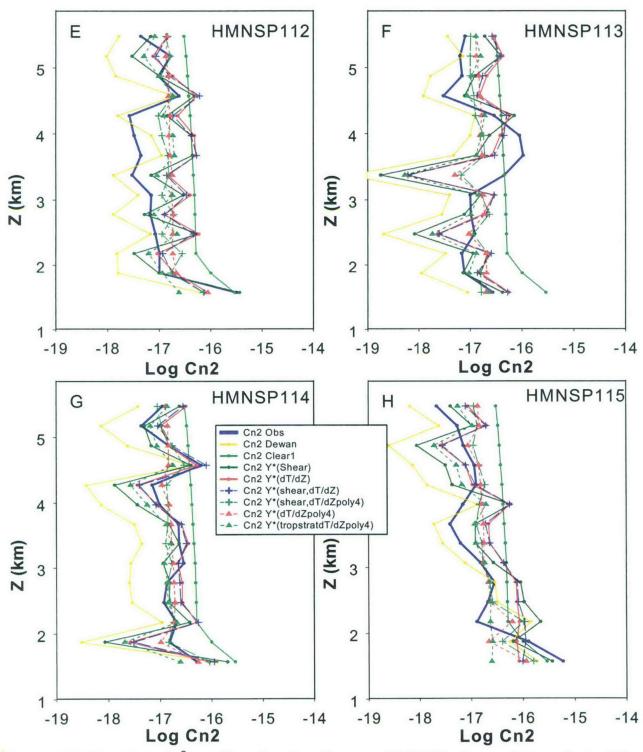


Figure 14 E - H. ${\rm C_n}^2$ profiles for the Dewan, CLEAR1, lower troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization, using the lower troposphere verification data set for thermosonde flights E. hmnsp112, F. hmnsp113, G. hmnsp114 and H. hmnsp115.

5.2 Troposphere and Stratosphere Parameterizations

5.2.1 Troposphere Parameterizations

The five troposphere Y* parameterizations (Equations (11a) - (11e)) shown in Table 6 generally perform better than the Dewan parameterization at diagnosing Y_{OBS}, but not as convincingly as the lower troposphere Y* parameterizations. Except for the y*shear parameterization, the Y* parameterizations perform modestly better than the Dewan parameterization. The y*dtdzpoly4 and y*sheardtdzpoly4 parameterizations performed the best, with a slight negative bias, markedly improved rms, and impressive correlation coefficients of 0.76 and 0.80 respectively. The Dewan parameterization performs much better in the troposphere than in the lower troposphere, with a much smaller bias, though the rms is larger than for most of the Y* parameterizations. As in the lower troposphere, the Dewan parameterization Y parameter correlates very poorly with the observed Y parameter, with a correlation coefficient of 0.08. v*tropstratdtdzpoly4 parameterization (Equation (13)) developed for the combined troposphere/stratosphere performs very similarly the v*dtdzpoly4 to y*sheardtdzpoly4 parameterizations developed for the troposphere, suggesting that a Y* parameterization developed for the combined troposphere/stratosphere may

Table 6. Mean, Bias, Root Mean Square Error, and Correlation Coefficient for the Dewan troposphere Y parameter, the troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization, using the troposphere verification data set.

Statistics	yobs	ydewan	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4	y*tropstratdtdzpoly4
Mean	2.09	1.97	1.86	1.99	1.99	2.04	2.03	2.04
BIAS		-0.12	-0.23	-0.09	-0.10	-0.05	-0.06	-0.05
RMS		0.98	0.99	0.78	0.75	0.64	0.60	0.65
Correlation		0.08	-0.08	0.63	0.68	0.76	0.80	0.74

adequately diagnose the Y parameter in the troposphere.

Figure 15 shows the distribution of Y_{OBS} plotted against the Y parameter calculated for the *y*sheardtdzpoly4*, Dewan, *y*sheardtdz* and *y*shear* parameterizations. The distribution of Y* values for the *y*sheardtdzpoly4* parameterization (Figure 15A) is superior to the other parameterizations, with most data points distributed along the diagonal line as desired. The distribution suggests a strong correlation between the *y*sheardtdzpoly4* Y parameter and Y_{OBS} as shown in Table 6. The *y*sheardtdzpoly4* parameterization has a noticeable negative bias for values of Y_{OBS} greater than about 3.5. The Y parameter values for the Dewan parameterization

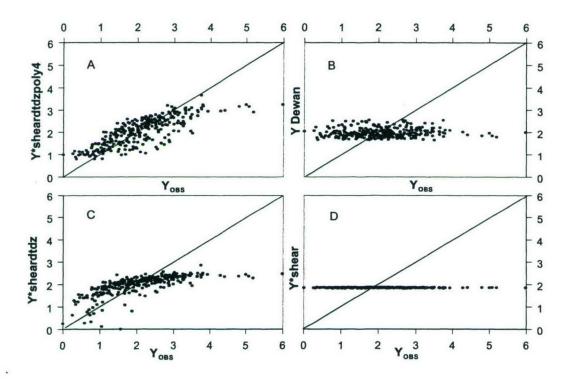


Figure 15. Distribution of Y_{OBS} and Y from the A. Y*sheardtdzpoly4, B. Dewan, C. Y*sheardtdz and D. Y*shear troposphere parameterizations for the troposphere verification data set.

(Figure 15B) range from about 1.5 to 2.5 while Y_{OBS} varies from 0.0 to 6.0. The distribution suggests minimal correlation between the Dewan Y parameter and Y_{OBS} as shown in Table 6. The Y* values for the *y*sheardtdz* parameterization (Figure 15C) show a tendency to follow the diagonal line for Y_{OBS} values from 0.0 to about 2.5, but fail to diagnose Y_{OBS} values greater than about 3.0. The distribution of the *y*shear* parameterization Y parameter (Figure 15D) shows little variation, with a mean value of about 1.9, indicating minimal correlation with the observed Y parameters.

In Table 7, all the Y* parameterizations except y*shear show modest improvement in diagnosing log C_n^2 compared to the Dewan parameterization. Once again, the y*dtdzpoly4 and the y*sheardtdzpoly4 parameterizations perform best. The CLEAR1 parameterization has the largest bias, 0.24, although the rms compares well with the y*dtdzpoly4 and y*sheardtdzpoly4 parameterizations, and CLEAR1 has the best correlation with the log C_n^2 observations, 0.51. As in Table 6 Y parameter comparisons, the y*tropstratdtdzpoly4 parameterization developed for the combined troposphere/stratosphere performs very similarly to the y*dtdzpoly4 and y*sheardtdzpoly4 parameterizations developed for the troposphere in diagnosing log

Table 7. Mean, Bias, Root Mean Square Error, and Correlation Coefficient of $\log {C_n}^2$ for the Dewan, CLEAR1, troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization using the troposphere verification data set.

Statistics	obs	dewan	clear1	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4	y*tropstratdtdzpoly4
Mean	-17.01	-17.13	-16.78	-17.25	-17.11	-17.11	-17.07	-17.09	-17.07
BIAS		-0.12	0.24	-0.23	-0.09	-0.10	-0.05	-0.07	-0.05
RMS		0.98	0.62	0.99	0.78	0.75	0.64	0.60	0.65
Correlation		0.25	0.51	0.16	0.22	0.30	0.32	0.44	0.31

 C_n^2 , suggesting that a Y* parameterization developed for the combined troposphere/stratosphere may adequately diagnose C_n^2 in the troposphere.

Figures 16 A and C show the distribution of the diagnosed Y parameter compared to the observed Y parameter for the hmnsp113 and hmnsp114 profiles. The

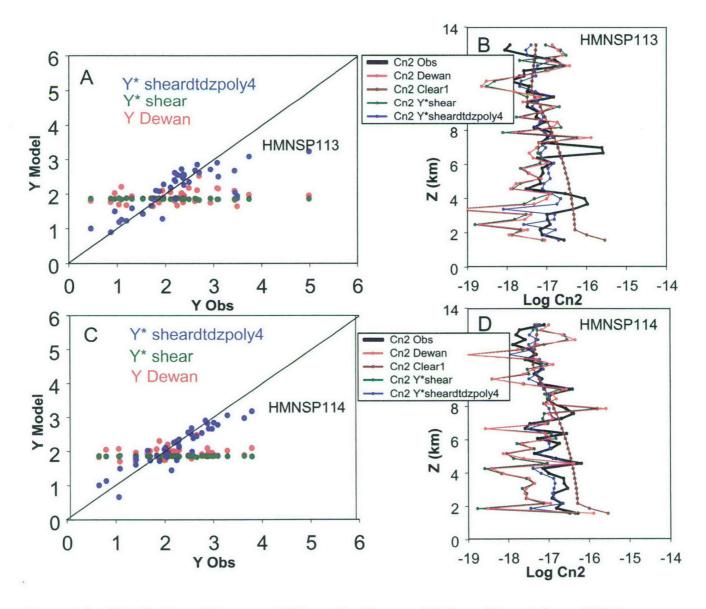


Figure 16. Distribution of Y_{OBS} and Y from the Dewan, Y^* sheardtdzpoly4, and Y^* shear troposphere parameterizations for the troposphere verification data set for thermosonde flights A. hmnsp113 and C. hmnsp114. C_n^2 profiles for the Dewan, CLEAR1, Y^* sheardtdzpoly4, and Y^* shear troposphere Y^* parameterizations for thermosonde flights B. hmnsp113 and D. hmnsp114.

 $y^*sheardtdzpoly4$ parameterization performs very well, with most Y* values falling very close to the diagonal line. The Dewan and y^*shear parameterizations' Y parameter values tend to be too high or too low, with only a few data points close to the diagonal line. This results in the C_n^2 profiles shown in Figures 16 B and D exhibiting overestimated maximum and underestimated minimum C_n^2 values. The $y^*sheardtdzpoly4$ C_n^2 profiles follow the observed profile quite well throughout the troposphere, though as in the lower troposphere, there is a tendency to underestimate the maximum and minimum C_n^2 values. The $y^*sheardtdzpoly4$ Y parameter distributions for the other profiles exhibit similar behavior following the diagonal, though most exhibit more spread about the diagonal.

Table 8 shows the vertically integrated C_n^2 values for the eight troposphere thermosonde profiles; Figure 17 displays the results graphically. The Dewan and CLEAR1 parameterizations appear to perform very well in diagnosing vertically integrated C_n^2 . The Dewan parameterization ranks first or second in six of eight profiles, while the CLEAR1 parameterization ranks first or second in four of eight profiles. The CLEAR1 parameterization has the smallest bias, the Dewan parameterization has the second smallest bias. The Dewan and CLEAR1 parameterizations rank second and third for rms. This result is somewhat unexpected since some of the Y* parameterizations, particularly the y*sheardtdzpoly4 parameterization, performed better at diagnosing the Y parameter and log C_n^2 . For the Dewan parameterization, part of this "success" may be due to it s tendency to overestimate maximums and underestimate minimums, with the vertically integrated C_n^2 value being close to the observed value. As an example, Figure 17 shows that the

Dewan parameterization diagnoses the vertically integrated C_n^2 value for the eighth profile (hmnsp115) very well. In Figure 18 H, the hmnsp115 C_n^2 profiles, the Dewan parameterization tends to overestimate C_n^2 maximums and underestimate C_n^2 minimums more than the $y^*sheardtdzpoly4$ parameterization, yet outperforms the $y^*sheardtdzpoly4$ parameterization. The vertically integrated C_n^2 value is much more difficult to evaluate than the Y parameter and log C_n^2 and probably requires many more profiles to properly evaluate parameterization performance. Note that the CLEAR1 parameterization also performs very well in diagnosing the vertically integrated C_n^2 value nearly exactly for three profiles (profiles 4, 6 and 8 in Figure 17), demonstrating its value as a climatological C_n^2 parameterization.

The Y* parameterizations all demonstrate a negative bias in diagnosing vertically integrated C_n^2 . Except for the *y*shear* parameterization, they all substantially underestimate the vertically integrated C_n^2 value for profiles 2 and 3 in Figure 17. As described in Section 5.1, parameterizations developed for the troposphere produce a negative bias for the Y parameter and $\log C_n^2$ when applied in the lower troposphere. Note that in Table 5 and Figure 13, the *y*sheardtdzpoly4* profile 3 produces a vertically integrated C_n^2 value of 3.52E-13 for the 1.572 km to 5.472 km lower troposphere. In Table 8 and Figure 17, the *y*sheardtdzpoly4* profile 3 produces a vertically integrated C_n^2 value of only 2.42E-13 for the entire troposphere, 1.572 km to 12.972 km. This strongly suggests the need for distinct parameterizations for the lower troposphere and the rest of the troposphere. Again, to properly evaluate parameterization performance in diagnosing vertically integrated C_n^2 probably requires a data set with many more profiles.

parameterizations, and the combined troposphere/stratosphere Y* parameterization, using the troposphere verification data set. Bias, Root Mean Square Error, and Correlation Coefficient of the vertically integrated C_n² profiles for each parameterization are shown. "+" indicates the parameterization performed better than the Dewan parameterization, "~" indicates the parameterization performed better than the CLEAR1 parameterization, "#" indicates the rank of the Vertically integrated Cn2 values for eight thermosonde profiles for the Dewan, CLEAR1, troposphere Y* parameterization for that profile. Table 8.

								The second secon	
	ops	dewan	clear1	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4	y*sheardtdz y*dtdzpoly4 y*sheardtdzpoly4 y*tropstratdtdzpoly4
hmnsp104	1.85E-13	3.19E-13 8	3.11E-13+7	2.14E-13+~3	1.92E-13+~1	hmnsp104 1.85E-13 3.19E-13 8 3.11E-13+ 7 2.14E-13+~3 1.92E-13+~1 1.93E-13+~2 1.22E-13+~5	1.22E-13+~5	1.17E-13+~6	1.35E-13+~4
hmnsp107	5.44E-13	hmnsp107 5.44E-13 3.73E-13 ~1 3.11E-13		2.40E-13 3	1.80E-13 5	2 2.40E-13 3 1.80E-13 5 1.84E-13 4 1.29E-13 8	1.29E-13 8	1.32E-13 7	1.36E-13 6
hmnsp108	1.07E-12	hmnsp108 1.07E-12 1.68E-12 ~2 3.11E-13	3.11E-13 3	5.89E-13+~1	1.64E-13 8	5.89E-13+~1 1.64E-13 8 1.80E-13 6 2.16E-13 5	2.16E-13 5	2.42E-13 4	1.72E-13 7
hmnsp111	2.98E-13	3.15E-13 2	3.11E-13+ 1	1.54E-13 3	1.18E-13 5	hmnsp111 2.98E-13 3.15E-13 2 3.11E-13+1 1.54E-13 3 1.18E-13 5 1.23E-13 4 1.07E-13 7	1.07E-13 7	1.02E-13 8	1.09E-13 6
hmnsp112	1.32E-13	3.69E-13 8	3.11E-13+7	1.61E-13+~6	1.31E-13+~1	hmnsp112 1.32E-13 3.69E-13 8 3.11E-13+ 7 1.61E-13+~6 1.31E-13+~1 1.41E-13+~2 1.15E-13+~5	1.15E-13+~5	1.17E-13+~4	1.18E-13+~3
hmnsp113	3.10E-13	hmnsp113 3.10E-13 1.25E-13 2 3.11E-13+	-	9.73E-14 8	1.10E-13 4	9.73E-14 8 1.10E-13 4 1.01E-13 7 1.05E-13 5	1.05E-13 5	1.04E-13 6	1.11E-13 3
hmnsp114	1.66E-13	hmnsp114 1.66E-13 1.95E-13 ~1 3.11E-13		1.26E-13 ~2	1.12E-13 ~5	8 1.26E-13 ~2 1.12E-13 ~5 1.16E-13 ~3 1.10E-13 ~6	1.10E-13~6	1.08E-13 ~7	1.13E-13 ~4
hmnsp115	3.35E-13	hmnsp115 3.35E-13 3.41E-13 ~1 3.11E-13		1.60E-13 5	1.61E-13 4	2 1.60E-13 5 1.61E-13 4 1.97E-13 3 1.12E-13 8	1.12E-13 8	1.34E-13 6	1.23E-13 7
BIAS		8.36E-14 2	-6.92E-14+ 1	-1.63E-13 3	-2.34E-13 5	8.36E-14 2 -6.92E-14+1 -1.63E-13 3 -2.34E-13 5 -2.26E-13 4 -2.54E-13 8	-2.54E-13 8	-2.48E-13 6	-2.53E-13 7
RMS		2.50E-13 ~2 2.97E-13		2.31E-13+~1	3.65E-13 7	3 2.31E-13+~1 3.65E-13 7 3.58E-13 5 3.61E-13 6	3.61E-13 6	3.52E-13 4	3.72E-13 8
Correlation	1.00	0.91 4	00.00	0.92+3	0.35 7	0.38 6	0.92+2	0.93+1	0.87 5

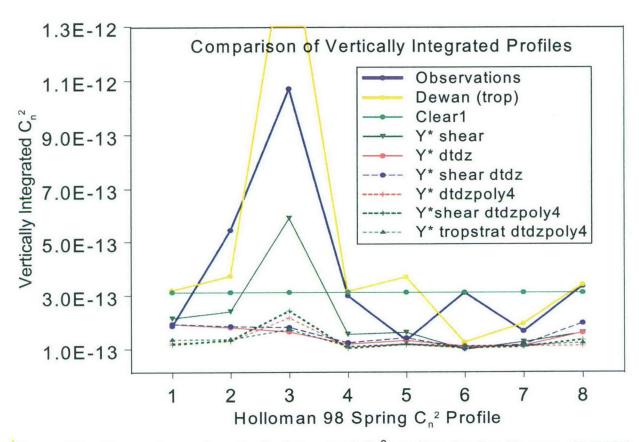


Figure 17. Comparison of vertically integrated C_n^2 profiles for the Dewan, CLEAR1, troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization using the troposphere verification data set. The numbers 1 thru 8 on the x-axis correspond to the thermosonde flights hmnsp104 thru hmnsp115 displayed in Table 8.

Figure 18 displays the troposphere C_n^2 profiles for each observed thermosonde sounding and for each parameterization. As in the lower troposphere there is a wide range of skill displayed in individual C_n^2 profiles. In Figure 18 E, the $y^*sheardtdzpoly4$ parameterization C_n^2 profile follows the observed profile quite well, although it underestimates many maximum C_n^2 peaks. In Figure 18 E, the Dewan parameterization C_n^2 profile exhibits many feature matches with the observed C_n^2 profile, but often significantly over overestimates maximums and underestimates

minimums. The Dewan and y*sheardtdzpoly4 parameterizations both perform poorly in diagnosing the observed C_n^2 profile in Figure 18 C up to about 8 km. Above 8 km the y*sheardtdzpoly4 parameterization performs very well while the Dewan parameterization diagnoses C_n^2 values that are significantly larger than the observed C_n^2 values. In general, the y*sheardtdzpoly4 parameterization follows the observed C_n^2 profile closer than the Dewan parameterization, although it underestimates the maximum values of observed C_n^2 , while the Dewan parameterization often significantly overestimates maximum C_n^2 values and underestimates minimum C_n^2 values.

The y*tropstratdtdzpoly4 parameterization, developed to test the usefulness of a single parameterization for the combined troposphere/stratosphere, performs well statistically in diagnosing the Y parameter and log C_n^2 in the troposphere. Its profiles tend to be similar, although smoother than the y*sheardtdzpoly4 profiles, and could potentially be useful as a single parameterization for the combined troposphere/stratosphere.

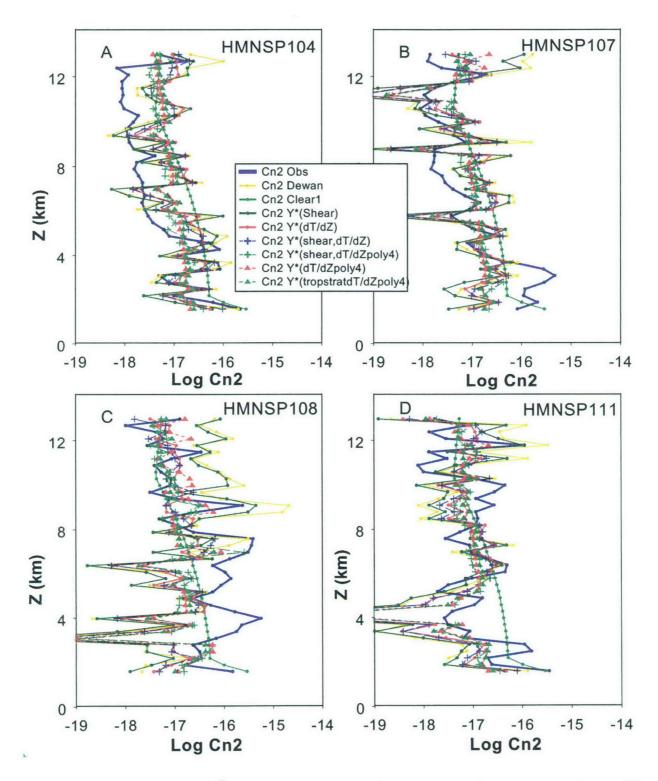


Figure 18 A - D. ${\rm C_n}^2$ profiles for the Dewan, CLEAR1, troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization, using the troposphere verification data set for thermosonde flights A. hmnsp104, B. hmnsp107, C. hmnsp108 and D. hmnsp111.

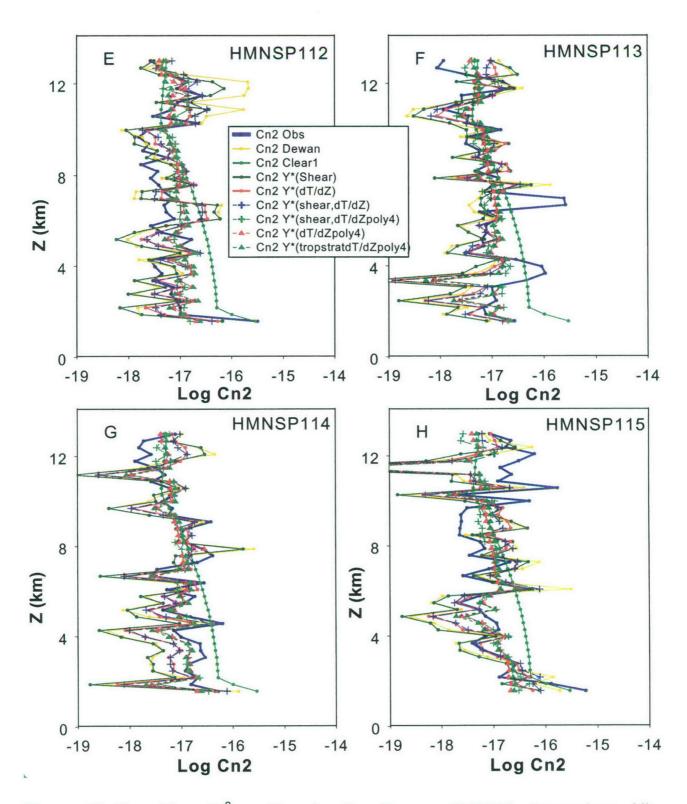


Figure 18 E - H. ${\rm C_n}^2$ profiles for the Dewan, CLEAR1, troposphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization, using the troposphere verification data set for thermosonde flights E. hmnsp112, F. hmnsp113, G. hmnsp114 and H. hmnsp115.

5.2.2 Stratosphere Parameterizations

Except for the y*shear parameterization (Equation (12a)), the Y* parameterizations (Equations (12b) - (12e)) perform better than the Dewan parameterization in diagnosing the Y parameter in the stratosphere, as shown in Table The Y* parameterizations have moderately smaller bias and rms values and 9. significantly improved correlation coefficients. Note that the mean observed Y parameter in the stratosphere is 0.91 compared to 2.62 in the lower troposphere and 2.09 in the entire troposphere. The decrease in the observed Y parameter is expected since C_n² typically decreases dramatically with height by at least several orders of magnitude, and according to the Dewan parameterization, C_n^2 is a function of 10^Y as shown in Equation (2). Thus, the mean observed Y parameter is expected to be smaller in the stratosphere than in the troposphere. The Dewan parameterization has a modest positive bias, 0.18, in the stratosphere, while the best Y* parameterizations have a negative bias of -0.06. The Dewan Y parameter correlates poorly with the observed Y parameter with a correlation coefficient of 0.13, while the correlation coefficients for the Y* models range from 0.50 to 0.66. As in the troposphere, the y*tropstratdtdzpoly4 parameterization (Equation (13)) developed for the combined troposphere/stratosphere

Table 9. Mean, Bias, Root Mean Square Error, and Correlation Coefficient for the Dewan stratosphere Y parameter, the stratosphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization, using the stratosphere verification data set.

Statistics	yobs	ydewan	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4	y*tropstratdtdzpoly4
Mean	0.91	1.09	0.74	0.78	0.79	0.86	0.86	0.84
BIAS		0.18	-0.17	-0.13	-0.12	-0.06	-0.06	-0.07
RMS		0.74	0.67	0.60	0.59	0.51	0.50	0.53
Correlation		0.13	0.13	0.50	0.53	0.64	0.66	0.60

performs very similarly to the *y*dtdzpoly4* and *y*sheardtdzpoly4* parameterizations developed for the stratosphere, suggesting that a Y* parameterization developed for the combined troposphere/stratosphere may adequately diagnose the Y parameter in the stratosphere.

The distribution of Y_{OBS} for the stratosphere shown in Figure 19 ranges from 0.0 to about 2.5, compared to 1.0 to 5.0 for the lower troposphere (Figure 10) and 0.0 to 5.0 for the entire troposphere (Figure 15). The distribution of the Y* values for the

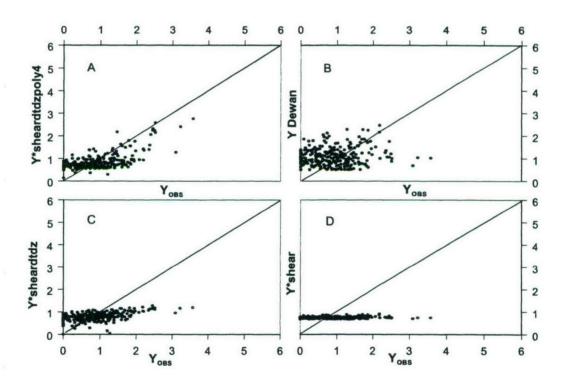


Figure 19. Distribution of Y_{OBS} and Y from the A. Y*sheardtdzpoly4, B. Dewan, C. Y*sheardtdz and D. Y*shear stratosphere parameterizations for the stratosphere verification data set.

y*sheardtdzpoly4 parameterization (Figure 19A) shows a tendency to follow the diagonal line, but many data points are concentrated near the y*sheardtdzpoly4 value of

about 0.9. The distribution of the Y* parameter for the y*sheardtdzpoly4 parameterization in the stratosphere is the best of those shown in Figure 19, but it suggests far less skill than the distributions for the lower troposphere (Figure 10A) and the entire troposphere (Figure 15A). The three other parameterizations show little or no tendency to be distributed along the diagonal line. These plots suggest the Y* parameterizations may not perform as well in the stratosphere as in the lower troposphere and the troposphere.

In Table 10, all the Y* parameterizations except y*shear show modest improvement compared to the Dewan parameterization in diagnosing $\log C_n^2$. Once again, the y*dtdzpoly4 and the y*sheardtdzpoly4 parameterizations perform best. The CLEAR1 parameterization has the largest bias, 0.20, though the rms compares well with the y*dtdzpoly4 and y*sheardtdzpoly4 parameterizations. The Dewan parameterization has the next highest bias, 0.19, and the largest rms, 0.76. CLEAR1, the y*dtdzpoly4 and the y*sheardtdzpoly4 parameterizations have the best correlation with the $\log C_n^2$ observations, 0.76. Once again, the y*tropstratdtdzpoly4 parameterization developed for the combined troposphere/stratosphere performs very similarly to the y*dtdzpoly4 and y*sheardtdzpoly4 parameterizations developed for the

Table 10. Mean, Bias, Root Mean Square Error, and Correlation Coefficient of $\log C_n^2$ for the Dewan, CLEAR1, stratosphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization using the stratosphere verification data set.

Statistics	Obs	dewan	clear1	y*shear	y*dtdz	y*sheardtdz	y*dtdzpoly4	y*sheardtdzpoly4	y*tropstratdtdzpoly4
Mean	-18.16	-17.97	-17.96	-18.32	-18.27	-18.27	-18.20	-18.20	-18.22
BIAS		0.19	0.20	-0.16	-0.11	-0.11	-0.04	-0.04	-0.06
RMS		0.76	0.58	0.69	0.62	0.61	0.53	0.53	0.56
Correlation		0.62	0.76	0.61	0.66	0.68	0.76	0.76	0.73

stratosphere in diagnosing $\log C_n^2$, suggesting that a Y* parameterization developed for the combined troposphere/stratosphere may adequately diagnose C_n^2 in the stratosphere.

Figure 20 A shows the distribution of the diagnosed Y parameter compared to the observed Y parameter for the hmnsp114 profile. The Dewan and y*shear parameterizations' Y parameter values tend to be too high or too low, with only a few data points close to the diagonal line, though over a much smaller range of Y values than for the lower troposphere and troposphere. This smaller departure from the observed Y value limits the significant overestimation of maximums and underestimation of minimums observed in the lower troposphere and troposphere profiles. There is a noticeable tendency for the Dewan parameterization ${\rm C_n}^2$ profile shown in Figure 20 B to deviate less from the observed C_n² profile than in the lower troposphere and troposphere profiles. The y*sheardtdzpoly4 parameterization, unlike the lower troposphere and troposphere Y* distributions which showed considerable tendency to fall along the diagonal line, shows only minimal tendency for Y* values to fall along the diagonal line. The y*sheardtdzpoly4 Y* values show less scatter about the diagonal than the Dewan Y values as the statistics suggest in Table 9. y*sheardtdzpoly4 C_n² profile shown in Figure 20 B follows the observed profile quite well throughout the stratosphere, although as in the lower troposphere and troposphere. there is a tendency to underestimate the maximum and minimum C_n² values. The y*sheardtdzpoly4 Y parameter distributions for the other profiles exhibit similar behavior showing only minimal tendency to fall along the diagonal, though showing less scatter than the Dewan parameterization Y values.

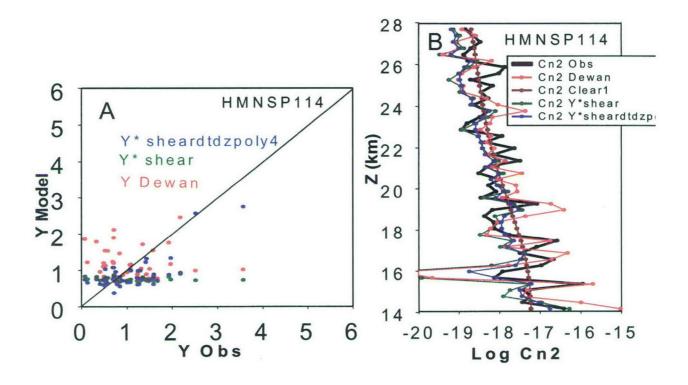


Figure 20. A. Distribution of Y_{OBS} and Y from the Dewan, Y^* sheardtdzpoly4, and Y^* shear stratosphere parameterizations for the stratosphere verification data set for thermosonde flight hmnsp114. B. C_n^2 profiles for the Dewan, CLEAR1, Y^* sheardtdzpoly4, and Y^* shear stratosphere Y^* parameterizations for thermosonde flight hmnsp114.

Table 11 shows the vertically integrated C_n^2 values for the six stratosphere thermosonde profiles; Figure 21 displays the results graphically. The CLEAR1 parameterization performs very well in diagnosing vertically integrated C_n^2 , ranking first or second in 5 of the 6 profiles. The CLEAR1 parameterization has the smallest bias and rms. The CLEAR1 parameterization is clearly very useful as a climatological C_n^2 parameterization in the stratosphere. The y*sheardtdzpoly4 parameterization ranks second or third in five of the six profiles and ranks third for bias and rms. All the Y*

parameterizations exhibit a relatively small bias and rms, slightly larger than for the parameterization. All the Y* parameterizations and the CLEAR1 parameterization perform better than the Dewan parameterization for the integrated C_n² values (except for profile 2), bias and rms. The Dewan parameterization has a noticeable positive bias, overestimating the integrated C_n² values, consistent with the positive bias for the Y parameter and log C_n² in the stratosphere, while the Y* and the CLEAR1 parameterizations have a smaller bias as is shown in Table 11 and Figure 21. It is important to note that the vertically integrated ${\rm C_n}^2$ values shown in Figure 21 for the stratosphere are much smaller than those shown in Figure 13 for the lower troposphere and Figure 17 for the troposphere. All points plotted for all models in Figure 21, except the fifth and sixth Dewan parameterization data points, would be contained between the x axis and 1.0 E-13 on Figure 17 for the troposphere vertically integrated C_n² values. Although the CLEAR1 and Y* parameterizations appear to perform very well compared to the Dewan parameterization in the stratosphere, the vertically integrated C_n² value is much more difficult to evaluate than the Y parameter and log C_n² and probably requires many more profiles to properly evaluate parameterization performance.

Figure 22 displays the stratosphere C_n^2 profiles for each observed thermosonde sounding and for each parameterization. In the stratosphere, deviations in the observed and parameterized C_n^2 values from the CLEAR1 parameterization C_n^2 values tend to be smaller than in the lower troposphere and troposphere. The Dewan parameterization tends to match observed features fairly well, although it tends to overestimate maximum C_n^2 values. The y*sheardtdzpoly4 parameterization tends to smooth out many features, but deviates less from the observed profile.

parameterizations, and the combined troposphere/stratosphere Y* parameterization using the stratosphere verification data set. Bias, Root Mean Square Error, and Correlation Coefficient of the vertically integrated C_n² profiles for each parameterization for that profile. indicates the parameterization performed better than the CLEAR1 parameterization, "#" indicates the rank of the parameterization are shown. "+" indicates the parameterization performed better than the Dewan parameterization, "~" Table 11. Vertically integrated Cn2 values for six thermosonde profiles for the Dewan, CLEAR1, stratosphere Y*

			_	_	_	_	_	_	
Correlation 1.00	RMS	BIAS	mnsp115	nmnsp114	nmnsp112	nmnsp111	nmnsp107	nmnsp104	
			8.02E-14	7.41E-14	7.19E-14	2.79E-14	3.63E-14	3.30E-14	obs
0.71 1	2.22E-13 8	1.46E-13 8	5.68E-13 8	2.96E-13 8	7.65E-14 ~1	6.82E-14 8	7.97E-14 8	1.09E-13 8	dewan
0.00	3.51E-14+ 1	-2.75E-14+ 1	2.64E-14+ 4	2.64E-14+ 1	2.64E-14 2	2.64E-14+ 1	2.64E-14+ 1	2.64E-14+ 1	clear1
0.70 2	3.52E-14+ 2	-3.10E-14+ 2	4.17E-14+~1	2.37E-14+ 2	2.05E-14 4	2.01E-14+2	1.50E-14+ 6	1.64E-14+ 5	y*shear
0.18 7	2.22E-13 8 3.51E-14+ 1 3.52E-14+ 2 4.35E-14+ 7 4.21E-14+ 6 3.96E-14+ 4	1.46E-13 8 -2.75E-14+ 1 -3.10E-14+ 2 -3.78E-14+ 7 -3.62E-14+ 6 -3.44E-14+ 4	hmnsp115 8.02E-14 5.68E-13 8 2.64E-14+ 4 4.17E-14+~1 1.71E-14+ 7 1.75E-14+ 6 2.81E-14+~3	hmnsp114 7.41E-14 2.96E-13 8 2.64E-14+ 1 2.37E-14+ 2 1.55E-14+ 7 1.93E-14+ 4 1.72E-14+ 5	hmnsp112 7.19E-14 7.65E-14 ~1 2.64E-14 2 2.05E-14 4 1.67E-14 8 1.73E-14 7 2.00E-14 5	hmnsp111 2.79E-14 6.82E-14 8 2.64E-14+ 1 2.01E-14+ 2 1.82E-14+ 7 1.88E-14+ 6 1.91E-14+ 5	hmnsp107 3.63E-14 7.97E-14 8 2.64E-14+ 1 1.50E-14+ 6 1.44E-14+ 7 1.63E-14+ 3 1.62E-14+ 4	hmnsp104 3.30E-14 1.09E-13 8 2.64E-14+ 1 1.64E-14+ 5 1.48E-14+ 7 1.66E-14+ 3 1.65E-14+ 4	y*dtdz
0.27 6 0.60 4	4.21E-14+6	-3.62E-14+6	1.75E-14+6	1.93E-14+ 4	1.73E-14 7	1.88E-14+6	1.63E-14+ 3	1.66E-14+ 3	y*sheardtdz
0.60 4	3.96E-14+4	-3.44E-14+ 4	2.81E-14+~3	1.72E-14+ 5	2.00E-14 5	1.91E-14+ 5	1.62E-14+ 4	1.65E-14+ 4	y*dtdzpoly4
0.69 3	3.82E-14+3	-3.30E-14+3	2.93E-14+~2	2.01E-14+3	2.06E-14 3	1.95E-14+ 4	1.78E-14+ 2	1.80E-14+ 2	y*sheardtdzpoly4
0.45 5	4.17E-14+5	-3.61E-14+ 5	2.11E-14+5	1.69E-14+ 6	1.85E-14 6	1.97E-14+3	1.53E-14+ 5	1.54E-14+ 6	y*sheardtdz y*dtdzpoly4 y*sheardtdzpoly4 y*tropstratdtdzpoly4

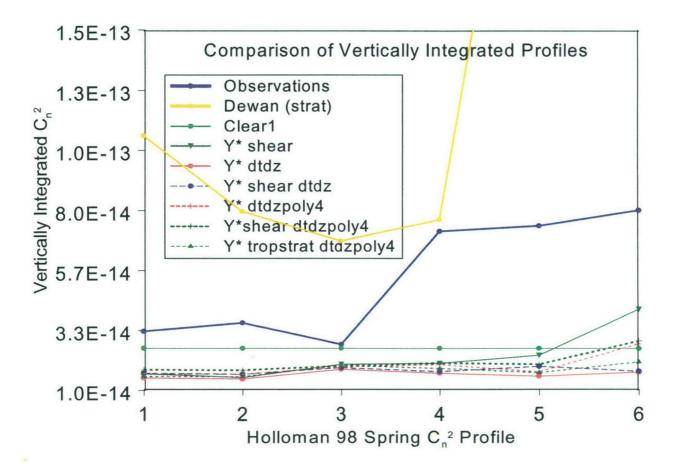


Figure 21. Comparison of vertically integrated C_n^2 profiles for the Dewan, CLEAR1, stratosphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization using the stratosphere verification data set. The numbers 1 thru 6 on the x-axis correspond to the thermosonde flights hmnsp104 thru hmnsp115 displayed in Table 11.

In Figure 22 A, the Dewan parameterization matches the location of peak C_n^2 values at 14.5 km, 18 km, 19.5 km, 21 km, 22 km, 22.5 km and others quite well, but each peak C_n^2 value is overestimated. The y*sheardtdzpoly4 parameterization matches the location of some of those features (14.5 km, 18 km, 19.5 km, 21 km, 22.5 km) but tends to underestimate the peak C_n^2 value or diagnose the peak C_n^2 value closely. In general, the y*sheardtdzpoly4 parameterization follows the observed C_n^2 profile more

closely than the Dewan parameterization, although it underestimates the maximum values of observed $C_n^{\ 2}$.

The y*tropstratdtdzpoly4 parameterization, developed to test the usefulness of a single parameterization for the combined troposphere/stratosphere, performs well statistically in diagnosing the Y parameter and log C_n^2 in the stratosphere. Its profiles tend to be very similar to the y*sheardtdzpoly4 profiles in the stratosphere, and could potentially be useful as a single parameterization for the combined troposphere/stratosphere.

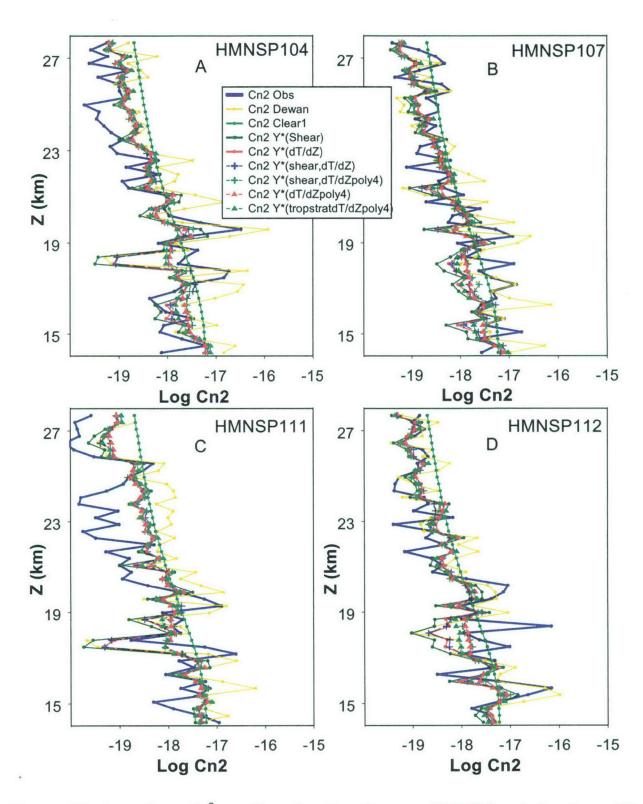


Figure 22 A - D. ${\rm C_n}^2$ profiles for the Dewan, CLEAR1, stratosphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization using the stratosphere verification data set for thermosonde flights A. hmnsp104, B. hmnsp107, C. hmnsp111, and D. hmnsp112.

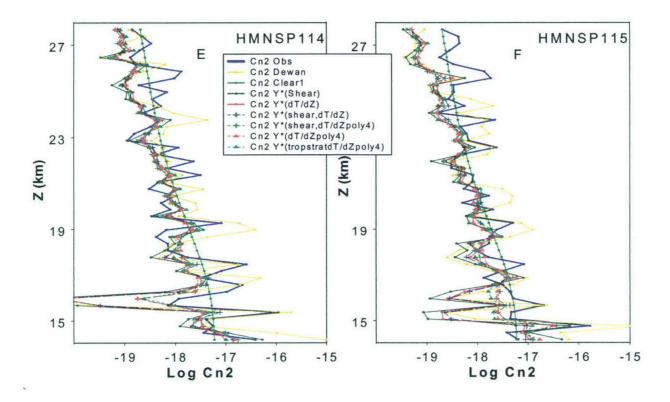


Figure 22 E - F. ${\rm C_n}^2$ profiles for the Dewan, CLEAR1, stratosphere Y* parameterizations, and the combined troposphere/stratosphere Y* parameterization using the stratosphere verification data set for thermosonde flights E. hmnsp114, and F. hmnsp115.

6 SUMMARY

The Dewan optical turbulence parameterization has been the Air Force Research Laboratory optical turbulence model of choice for various research efforts involving optical propagation during the past several years, providing useful vertical profiles of C_n^2 in the upper troposphere and stratosphere. However, the statistical relationships relating the Dewan Y parameter to wind shear, which form the basis of the Dewan parameterization, are often not found in atmospheric measurement data.

The Dewan parameterization defines the Y parameter as a linear function of wind shear, although scatter plots showing the distribution of Y_{OBS} and wind shear do not show a relationship. Scatter plots showing the distribution of Y_{OBS} and temperature lapse rate suggest a nonlinear relationship with decreasing values of Y_{OBS} as lapse rate increases. A fourth order polynomial was determined to be the best regression fit describing the nonlinear relationship present in the Y_{OBS} and temperature lapse rate observations. Several parameterizations were developed for each of the lower troposphere, the troposphere, and the stratosphere, describing the observed Y parameter as a linear function of wind shear, a linear function of temperature lapse rate, a fourth order polynomial function of lapse rate, and a combined linear function of wind shear and fourth order polynomial function of lapse rate.

In the lower troposphere, the Y* parameterizations significantly outperform the Dewan parameterization in diagnosing the Y parameter, $\log C_n^2$, and vertically integrated C_n^2 . Accurately diagnosing the Y parameter is essential to improving the diagnosis of C_n^2 . The scatter plot showing the distribution of Y_{OBS} plotted against the y*sheardtdzpoly4 parameterization Y parameter shows a strong correlation between the

y*sheardtdzpoly4 Y parameter and Y_{OBS} and is a dramatic improvement over the Dewan Y parameter distribution.

The Dewan parameterization demonstrates a consistent negative bias in diagnosing C_n^2 in the lower troposphere, which would result in a substantial underestimate of optical turbulence effects. The y*sheardtdzpoly4 parameterization describing Y* as a fourth order polynomial function of temperature lapse rate and a linear function of wind shear performed particularly well.

Y* parameterizations developed for the troposphere were also tested using the lower troposphere verification data set to determine if the parameterizations developed specifically for the lower troposphere performed better in the lower troposphere than parameterizations developed for the entire troposphere. The results show that parameterizations developed specifically for the lower troposphere did perform considerably better in the lower troposphere than parameterizations developed for the entire troposphere.

In the troposphere, the Y* parameterizations perform modestly better than the Dewan parameterization in diagnosing the Y parameter and $\log C_n^2$. As in the lower troposphere, the scatter plot showing the distribution of Y_{OBS} plotted against the y*sheardtdzpoly4 parameterization Y parameter shows a strong correlation between the y*sheardtdzpoly4 Y parameter and Y_{OBS} and is a significant improvement over the Dewan Y parameter distribution. In general, the y*sheardtdzpoly4 parameterization follows the observed C_n^2 profiles more closely than the Dewan parameterization, although it underestimates the maximum values of observed C_n^2 , while the Dewan

parameterization often significantly overestimates maximum C_n^2 values and underestimates minimum C_n^2 values.

Statistically, the Dewan parameterization performs better than the Y* parameterizations in diagnosing vertically integrated C_n^2 in the troposphere. This result is somewhat unexpected since some of the Y* parameterizations, particularly the y*sheardtdzpoly4 parameterization, perform better at diagnosing the Y parameter and log C_n^2 . Close examination of the Dewan parameterization C_n^2 profiles suggests that part of this "success" may be due to its tendency to overestimate maximums and underestimate minimums, with the vertically integrated C_n^2 value being fortuitously close to the observed value. The vertically integrated C_n^2 value is much more difficult to evaluate than the Y parameter and log C_n^2 and probably requires many more profiles to adequately evaluate parameterization performance.

The Y* parameterizations all demonstrate a negative bias in diagnosing vertically integrated C_n^2 in the troposphere. Tests performed for the lower troposphere show that parameterizations developed for the entire troposphere produce a negative bias for the Y parameter and $\log C_n^2$ when applied in the lower troposphere. This strongly suggests the need for distinct parameterizations for the lower troposphere and the rest of the troposphere, which would help to eliminate the negative bias in diagnosing vertically integrated C_n^2 in the troposphere.

In the stratosphere, the Y* parameterizations modestly outperform the Dewan parameterization in diagnosing the Y parameter, $\log C_n^2$ and vertically integrated C_n^2 . The scatter plot showing the distribution of Y_{OBS} plotted against the *y*sheardtdzpoly4* parameterization Y parameter shows less skill than for the lower troposphere and the

troposphere, although it is certainly superior to the Dewan Y parameter distribution. In the stratosphere, the Dewan parameterization tends to match observed features fairly well, although it tends to overestimate maximum C_n^2 values, resulting in the fairly large positive bias in diagnosing vertically integrated C_n^2 . The $y^*sheardtdzpoly4$ parameterization tends to smooth out many features, but deviates less from the observed profile than the Dewan parameterization, resulting in a slight negative bias in diagnosing vertically integrated C_n^2 .

The $y^*tropstratdtdzpoly4$ parameterization, developed to test the usefulness of a single parameterization for the combined troposphere/stratosphere, performs well statistically in diagnosing the Y parameter and log C_n^2 in the troposphere and the stratosphere. Its profiles tend to be very similar to the $y^*sheardtdzpoly4$ profiles and could potentially be useful as a single parameterization for the combined troposphere/stratosphere. However, tests performed for the lower troposphere suggest that this parameterization will likely produce a negative bias for the Y parameter and log C_n^2 in the lower troposphere.

The CLEAR1 parameterization performs very well in diagnosing vertically integrated $C_n^{\,2}$ in both the troposphere and the stratosphere, clearly demonstrating that it is a very useful climatological optical turbulence parameterization.

The modified-Dewan optical turbulence parameterizations described in this study were developed and tested using optical turbulence data from one season, Spring, and for one location, Holloman, New Mexico. The performance of the parameterizations in other seasons or locations is untested.

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